CREW PROCEDURES ORBITAL GUIDANCE AND NAVIGATION PROGRAM

NAVIGATION SECTION

1.0 PROGRAM DESCRIPTION

The CPB Guidance and Navigation Program, henceforth referred to as AAP, is a modification set to Program BETELGEUSE. The program library currently exists on tapes 183 or 184 in the Building 35 Tape Library.

AAP provides environment and estimated states for two vehicles, rendezvous sensors and an inertial platform. These vehicles may be in orbit about either the earth or moon.

The navigation filter is a generalization of the Apollo-LM square-root filter to two-vehicle estimation plus estimated sensor biases. A detailed exposition of the theory of this filter may be found in references 1 and 2, and of the covariance advancement method in reference 3. The filter accepts measurements of relative angle, range and/or range-rate, and updates the state of either or both vehicles in an optimal fashion.

The program is structured into a main overlay, and 1st and 2nd primary overlays. The main overlay lists input/output devices, zeros working core and sets values for sensor error models. The 1st primary overlay contains input/output routines, state and covariance integrators and the navigation package. The 2nd primary overlay contains subroutines necessary to compute rendezvous maneuvers as currently defined.

By the setting of appropriate flags, AAP can be caused to run in either single-run or monte-carlo modes. Data as specified by the user is collected at intervals on each cycle of a monte-carlo run, and stored on a local mass-storage file for later transference to a permanent data tape. This data may then be processed statistically by a separate program.

The following sections will deal with those portions of AAP which constitute the navigation function, and its controlling subroutines. It will be assumed that the user is otherwise familiar with standard BETELGEUSE functions and the operation of the guidance overlay.

2.0 PROGRAM CONTROL

The sequence of cards portrayed in figure 2.1 constitutes the controlling set to read the program library, update corrections, load, execute and transfer run data from local to permanent storage. The following notes corresponding to cards identified with the same number are provided for clarity:

1. Creates local mass-storage file for monte-carlo data

2. Requests program library tape

3. Copies program library to local file

4. Copies compile file to local file

5. Update changed subroutines from program library

6. Compile updated routines

7. Rewind update file

8. Copy changed subroutines into compile file

9. Request additional field length for loading

10. Load updated program

11. Set up overlay linkages

12. Reduce to execution field length

13. Execute

14. Create a dummy file

15. Rewind the program data storage file

16. Request the permanent data storage tape

17. Turn data storage tape past previously stored data

18. Copy new data onto tape

Should abnormal termination of the program occur, placement of an EXIT. card following UNLOAD(DTAPE) will cause control to be transferred to the EXIT. card, and execution of all cards following. The card sequence from REWIND(FAKE) to UNLOAD(DTAPE) should be duplicated and placed behind the EXIT. card. This will prevent loss of data in the event of abnormal termination. In order for a subroutine to be modified, it must already exist in the tape program library. Attempts to add a previously non-existent subroutine will cause error termination. To ameliorate the effect of this restriction, the first primary overlay contains five dummy subroutines, OPEN1 to OPEN5. These may be modified as required to provide currently undefined program operations.

3.0 REQUIRED INPUT DATA

In order to produce desired program operation, it is necessary to specify sensor and IU error models, navigation initializing and control information and platform alignment times. If maneuvers are to be performed, targeting data and instructions as to which will be stored for later processing are also required. Vehicle state vectors and their covariance are specified in the usual way for all BETELGEUSE programs requiring such information.

3.1 HARD-WIRED CONSTANTS

Rendezvous sensors, alignments and maneuver applications are all modelled by AAP as pure Gaussian processes. Values for the means and standard deviations of these processes are set in program MAIN of the main overlay by FORTRAN replacement statements. Figure 3.1 presents the relevant portions of program MAIN with the subject variables underlined. A correction set of similar replacement statements must be contained in the update portion of the operating deck if sensors being modelled have a different error model than that shown in figure 3.1:

VARS — ACCELEROMETER SCALE FACTOR ERROR (DIMENSIONLESS)

VARA — DELTA-V CUTOFF UNCERTAINTY (FT/SEC)

RVAR — ACTUAL VALUE OF RANGE MEASUREMENT ERROR AS A FRACTION OF TOTAL RANGE (DIMENSIONLESS)

RVARMIN — ACTUAL VALUE OF MINIMUM RANGE ERROR (FT)

VVAR — ACTUAL VALUE OF RANGE-RATE MEASUREMENT ERROR AS A FRACTION OF TOTAL RANGE-RATE (DIMENSIONLESS)

VVARMIN — ACTUAL VALUE OF MINIMUM RANGE-RATE ERROR (FT/SEC)

VARAZ — ACTUAL VALUE OF AZIMUTH MEASUREMENT ERROR (RADIANS)

VAREL — ACTUAL VALUE OF ELEVATION MEASUREMENT ERROR (RADIANS)

BR, BV, BAZ, BEL — ACTUAL VALUES OF RANGE, RANGE-RATE, AZIMUTH AND ELEVATION MEASUREMENT BIASES. THESE ARE SET IN THE FIRST VISIT TO THE SENSOR NOISE SUBROUTINE ON THE BASIS OF THE VALUES OF BRO, BVO, BAZO AND BELO

BRO — ACTUAL VALUE OF RANGE BIAS ERROR (FT)

BVO — ACTUAL VALUE OF RANGE-RATE BIAS ERROR (FT/SEC)

BAZO — ACTUAL VALUE OF AZIMUTH BIAS ERROR (RADIANS)

BELO — ACTUAL VALUE OF ELEVATION BIAS ERROR (RADIANS)

GDR — ACTUAL VALUE OF GYRO DRIFT RATE ERROR (RADIANS/SEC)

ALIGNB — ACTUAL VALUE OF INITIAL MIS-ALIGNMENT BIAS ERROR (RADIANS)

NFAMA — INITIALIZER FOR MANEUVER APPLICATION ERRORS

NFAMB — INITIALIZER FOR SENSOR BIAS ERRORS

```
03/08/73   MSC VERSION 9.0   SCOPE 3.3   296-4
13.30.57.0ARJH3S
13.30.57.0ARJH,CM60000,P27,T100 00,MT01.
13.30.57.REWIND(CAL)
13.30.57.REWIND(STAT)                              1.
13.30.57.REQUEST(TAPE1)REEL183                     2.
13.31.14.ROLLOUT COMPLETED.   (FL   60000)
14.33.51.ROLLIN COMPLETED.
14.34.04. (33 ASSIGNED)
14.34.04.REWIND(TAPE1)
14.34.04.COPYBF(TAPE1,KEN1)                        3.
14.34.16.COPYBF(TAPE1,KEN2)                        4.
14.34.25.UNLOAD(TAPE1)
14.34.26.REWIND(KEN1,KEN2)
14.34.26.UPDATE(P=KEN1,W)                          5.
14.34.27.READING INPUT
14.35.16.UPDATE COMPLETE
14.35.16.FTN(I=COMPILE)                            6.
14.36.30.     17.779 CP SECONDS COMPILATION TIME
14.36.31.REWIND(LGO)                               7.
14.36.31.COPYL(KEN2,LGO,XXX)                       8.
14.36.33. RK         UPDATED
14.36.34. INPUT      UPDATED
14.36.34. GNEXEC     UPDATED
14.36.34. DELTAV     UPDATED
14.36.34. STORE1     UPDATED
14.36.35. OUTDAT     UPDATED
14.36.37. POPOUT     UPDATED
14.36.39. OPEN3      UPDATED
14.36.40. OPEN4      UPDATED
14.36.40. OPEN5      UPDATED
14.36.42. CLOD       UPDATED
14.36.42. ICERR      UPDATED
14.36.43. GIDSEL     UPDATED
14.36.49.COPYL DONE
14.36.49.RFL,70000.                                9.
14.36.49.LOAD(XXX)                                 10.
14.37.26.NOGO.                                     11.
14.37.26.RFL,60000.                                12.
14.37.26.AAP.                                      13.
14.38.44.ROLLOUT COMPLETED.   (FL   60000)
14.52.30.ROLLIN COMPLETED.
15.27.52.EXIT
15.27.52.REWIND(FAKE)                              14.
15.27.52.REWIND(STAT)                              15.
15.27.52.REQUEST(DTAPE)        ENABLE WRITE ON REEL
15.27.52.285                                       16.
15.28.17. (32 ASSIGNED)
15.28.17.REWIND(DTAPE)
15.28.17.COPYBF(DTAPE,FAKE,2)                      17.
15.28.30.COPYBF(STAT,DTAPE)                        18.
15.29.13.RELEASE(FAKE)
15.29.13.UNLOAD(DTAPE)
15.29.14.MT 32 BLOCKS WRITTEN--000156
15.29.20.CP    1350.320 SEC.
15.29.20.PP     315.604 SEC.
15.29.20.IO     068.445 SEC.
```

Figure 2.1 AAP Control Cards

```
C       LOAD ERROR MODEL
        VARS=1.E-4
        VARA=1.E-1
C
          RVAR=0.
          RVARMIN = 33.
        VVAR=4.3E-3
        VVARMIN=.43
          VARAZ=2.E-3
          VAREL=2.E-3
        BR=0.
        BV=0.
        BAZ=0.
        BEL=0.
        BRO=0.
        BVO=0.
        BAZO=17.45E-3
        BELO=17.45E-3
C
        GDR=1.45E-7
        ALIGNB=3.E-4
        NFAMA= 46728
        NFAMB= 12944
        NFAMC= 31171
        NFAMV= 22222
C
          RVARB=0.
          RVARMNB = 33.
        VVARB=4.3E-3
        VVARMNB=.43
          VARAZB=2.E-3
          VARELB=2.E-3
C
```

Figure 3.1 Hard-wired constants

NFAMC INITIALIZER FOR PLATFORM MISALIGNMENT AND DRIFT ERRORS

NFAMV INITIALIZER FOR SENSOR RANDOM NOISE ERRORS

RVARB FILTER VALUE OF RANGE MEASUREMENT ERROR AS A FRACTION OF TOTAL RANGE (DIMENSIONLESS)

RVARMNB FILTER VALUE OF MINIMUM RANGE ERROR (FT)

VVARB FILTER VALUE OF RANGE-RATE MEASUREMENT ERROR AS A FRACTION OF TOTAL RANGE-RATE (DIMENSIONLESS)

VVARMNB FILTER VALUE OF MINIMUM RANGE-RATE ERROR (FT/SEC)

VARAZB FILTER VALUE OF AZIMUTH MEASUREMENT ERROR (RADIANS)

VARELB FILTER VALUE OF ELEVATION MEASUREMENT ERROR (RADIANS)

All values are to be given 1-sigma (standard deviation) including sensor biases. Bias processes are considered as having a zero mean; these are constructed at the beginning of program execution and become the mean value of any subsequent random process. To the extent that it models or ignores sensor biases, and compensates for random noise in the weighting process, the filter contains a model of every known random process affecting the value of a measurement. The difference between the actual value of a random process, and the filter value, is that in general the actual values can only be guessed at. Hence the actual and filter values are not in general the same, and it is customary to set the filter value larger than the largest expected value of the actual errors.

3.2 INPUT DATA CARDS

In addition to normal BETELGEUSE input cards, additional cards are defined which control the storage of data at selected maneuvers, the performance of navigation processes and the time of platform alignments. Also, some of the BETELGEUSE input features are utilized in the normal way to set flags and support the input of other required data. These will now be examined on a card-by-card basis as to placement and content:

CARD #1, INTEGER PARAMETER, 14I5

IDENT, C1-C5, Program will execute number of monte-carlo cycles equal to IDENT

NP, C6-C10, Number of P-array variables in data file

NINT, C11-C15, Starting value of NGUIDE (determines which maneuver of rendezvous sequence is first)

NFIRST, C16-C20, BETELGEUSE initial condition option

NTABLE, C21-C25, Number of tables in input data file

NV, C26-C30, Number of variables to be integrated (NV=37 for navigation runs)

NMORE, C31-C35, Number of additional integer parameters to be read in on succeeding cards

NT(1), C36-C40,

NCOL, C41-C45, Number of columns of W to be advanced (must be greater than or equal to the number of variables being estimated)

C46-C50, Not used

C51-C55, Not used

NPER, C56-C60, Number of navigation procedure cards in input file

IBLATE, C61-C65, Keplerian gravity flag. IBLATE=0 specifies point mass accelerations. IBLATE=1 causes computation of aspherical perturbations

IE, C66-C70, TPI angle search flag. IE=0 causes TPI to be done on time. Otherwise, angle.

CARD #2, CARD #3, HOLLERITH FIELD, 72H each
Two comment cards for the purpose of describing and labeling the run deck

CARD #4, DATA STORAGE CONTROL, 15I5
Flags for the storage of up to 8 sets state vector and delta-v information at selected rendezvous maneuvers. The number of the integer field in the sequence of 15 determines at which maneuvers data will be stored, by setting a right-adjusted 1 in that field. EXAMPLE: A '1' in column 25 of this card causes data to be stored at NGUIDE=5, that is, TPI.

CARD #5 et seq., BETELGEUSE FLOATING POINT (P-ARRAY), I5 ,E15.7
P(297)=Input value of NFAM2, initial state error initializer

NAVIGATION PROCESS CONTROL CARDS, CONTIGUOUS WITH END OF BETELGEUSE FLOATING POINT ENTRIES. THE NUMBER OF THESE CARDS MUST BE EQUAL TO NPER, AND CONVERSELY. 2I2,3F7.1,9F5.2,I2

The following explaination applies to Ith card (I=1,NPER), that is, the Ith navigation procedure to be executed during each run:

NE(I), C1-C2, Value of NE(I) dtermines when card will become active. If NE(I)=5, card will become active during guidance period when NGUIDE=5, i.e. preTPI.

NM(I), C3-C4, Value of NM(I) determines what type of measurement this card will cause to be taken:
NM(I)=1: Radar mark (R,RDOT, TWO ANGLES)
NM(I)=2: VHF mark (R ONLY)
NM(I)=3: OPTICS mark (TWO ANGLES ONLY)
NM(I)=4: SHUTTLE? (R, TWO ANGLES)

DTL(I), C5-C11, Number of minutes after previous maneuver that this card will become active. If there has been no previous maneuver, program will define previous maneuver as having occurred at time=0.

DTN(I), C12-C18, Number of minutes before next maneuver that this card will become inactive.

DTM(I), C19-C25, Number of minutes between measurements called for by this card.

USP(I), C26-C30, 1-sigma radius of S/C position uncertainty, in thousands of feet

USV(I), C31-C35, 1-sigma radius of S/C velocity uncertainty, in feet/second

UTP(I), C36-C40, 1-sigma radius of TGT position uncertainty, in thousands of feet

UTV(I), C41-C45, 1-sigma radius of TGT velocity uncertainty, in feet/second

SR(I), C46-C50, 1-sigma range measurement bias uncertainty, in thousands of feet

SRD(I), C51-C55, 1-sigma range-rate measurement bias uncertainty, in feet/second

SO(I), C56-C60, 1-sigma radius of angle measurement bias uncertainty about line-of-sight, milliradians

SC1(I) and SC2(I), C61-C70, Dummy estimated constants, not currently defined, corresponding to elements 17 and 18 of the estimated state vector. Unless defined, these should be set to 0.

NW(I), C71-C72, Reinitialization flag. NW(I)=0 causes a spherical reinitialization of the W-matrix to a diagonal form with values specified by C26-C70. NW(I)=1 inhibits the resetting of W and causes this procedure to become active with the existing value of W.

Figure 4.1 AAP Block Data

BETELGEUSE STATE 1320.0 A

	RAD VEC	LONGITUDE	LATITUDE	ALT RATE	HOR VEL	HEADING B	X-BAR	Y-BAR	Z-BAR	U-BAR	V-BAR	W-BAR
NAV	215.9674.	36.154	-75.881	-.98	25557.69	173.[illegible]48	127.	193.	-3.	-2.84	-4.49	.18
ACT	21513368.	36.138	-75.873	.92	25556.58	172.958	180.	286.	-13.	-2.73	-4.13	.40

CARTESIAN STATE 1320.0

	XS(BRF)	YS(BRF)	ZS(BRF)	XSD(BRF)	YSD(BRF)	ZSD(BRF)	XT(BRF)	YT(BRF)	ZT(BRF)	XTD(BRF)	YTD(BRF)	ZTD(BRF)
NAV	4242138.	-3088186.	-20859853.	12505.60	22276.39	-753.70	4242053.	-3088362.	-20859976.	12507.54	22280.53	-750.99
ACT	4240690.	-3096668.	-20862699.	12504.20	22275.49	-765.63	4240567.	-3096932.	-20862871.	12505.78	22279.45	-763.09

CARTESIAN STATE ERRORS IN MEASUREMENT FRAME

XSE(MF)	YSE(MF)	ZSE(MF)	XSDE(MF)	YSDE(MF)	ZSDE(MF)	XTE(MF)	YTE(MF)	ZTE(MF)	XTDE(MF)	YTDE(MF)	ZTDE(MF)
1041.	-8596.	2677.	-8.93	-7.27	-3.57	1047.	-8704.	2682.	-8.84	-7.62	-3.80

RELATIVE STATE ERRORS

==========================BIAS ESTIMATION========================*******************POSITION AND VELOCITY**************************

RANGE	R-RATE	AZIMUTH	ELEV	SC1	SC2	XRE(MF)	YRE(MF)	ZRE(MF)	XRDE(MF)	YRDE(MF)	ZRDE(MF)
0.0	0.000	10.347	6.307	0.000	0.00	5.	-108.	5.	.09	-.35	-.23

D

D-R	S-DOT	SD-COEL	DELTA-H	DELH-DOT
94.	-.326	-.234	52.	-.001

RELATIVE PARAMETERS

	RANGE	R-RATE	AZIMUTH	REL ELEV
NAV	231.	-5.31	6.944	33.793
ACT	339.	-4.96	4.483	32.152
MEAS.	301.	-4.43	1.574	1.395

MARKS C

MFLG	0	UR	17.5
RAD	17	URD	.0
VHF	0	UAZ	4.6
OPT	0	UEL	4.6
		SR	.883
		SRD	.997
		SAZ	.400
		SEL	.397

INERTIAL STANDARD DEVIATIONS E

S/C	5349.	8738.	4128.	9.36	11.71	2.81
TGT	5349.	8737.	4128.	9.36	11.71	2.81
BIAS	0.	.000	1.576	1.96	0.00	0.00

-.69 -4.62 1.70

43.5 RNG -.015839 ETADI -.689 DVNOR
-4.618 RNGRT .033067 BETADI -4.618 DVLOS
1.700 DVOOP

F

COVARIANCE OF RELATIVE ERRORS (MF)

X	.144E+01	.452E+01	.566E-01	.228E-01	.591E-02	.386E-03
Y	F1 .19554	.169E+02	.371E+01	.433E-01	.433E+00	.246E-01
Z	.02721	.15186	.145E+01	.454E-03	.629E-02	.225E-01
XD	.65958	.10667	.01308	.240E-01	.686E-04	.290E-05
YD	.15770	.92164	.15666	.10274	.278E-01	.574E-04
ZD	.01185	.06438	.68773	.00533	.09129	.226E-01

G

THE NAVIGATION BURN ESTIMATE IS DU= -2.412 DV= 4.344 DW= .109
THE ACTUAL COMPONENTS WERE DU= -2.484 DV= 4.294 DW= .187

BETELGEUSE STATE 1380.0

	RAD VEC	LONGITUDE	LATITUDE	ALT RATE	HOR VEL	HEADING	X-BAR	Y-BAR	Z-BAR	U-BAR	V-BAR	W-BAR
NAV	215.9450.	19.309	-75.796	-3.19	25561.83	-171.709	-31.	-56.	-12.	-.11	-.23	-.31
ACT	21513483.	19.404	-75.795	-1.09	25560.34	-171.810	15.	35.	22.	.07	-.06	.07

CARTESIAN STATE 1380.0

ALIGNMENT CONTROL CARD, CONTIGUOUS WITH THE END OF NAVIGATION PROCESS CONTROL CARDS. THIS CARD MUST BE PRESENT, EVEN IF BLANK. I2,10E7.1

Program will read up to 10 decimal fields on card, following integer field. Integer in first two columns determines how many F-fields will be read in subsequent columns. Subsequent columns contain the times, in seconds, of desired platform realignments during each run. If card is blank, an alignment is automatically performed at time=0.

INPUT STATE COVARIANCE MATRIX, IMMEDIATELY FOLLOWING ALIGNMENT CARD. THIS MATRIX MUST BE PRESENT WITH DIMENSION 24 x 24 IF P(297) IS OTHER THAN ZERO.

INPUT TABLES CALLED FOR BY NTABLE, IMMEDIATELY FOLLOWING INPUT COVARIANCE MATRIX. THESE TABLES MUST BE PRESENT IF NTABLE IS GREATER THAN ZERO.

4.0 OUTPUT

Program AAP originates printed and mass storage output. Printed output is originated by the 2nd primary overlay during maneuver computations, by the 1st primary overlay during maneuver applications, and at each platform alignment. Block data on all vehicle states, and the status of the navigation, is printed periodically as specified by the user. A sample of such output is shown in figure 4.1 and will be discussed below. Block data print interval is controlled by setting P(9) equal to the desired print interval, in seconds. Block data is automatically printed every time the guidance (2nd) overlay is called, or whenever the W-matrix is reinitialized.

Area A: Time, in seconds, of the block print.

Area B: Azimuth of ground track; east is 0°, south is 90°, etc.

Cartesian State: Earth centered inertial frame (BRF)

Cartesian State Errors in Measurement Frame (MF): Measurement frame is defined as follows-

$$\mathrm{UNIT}(\underline{X}_{MF})=\mathrm{UNIT}(\underline{Y}_{MF} \times \underline{Z}_{MF})$$

$$\mathrm{UNIT}(\underline{Y}_{MF})=\mathrm{UNIT}(\underline{R}_{TGT} - \underline{R}_{S/C})$$

$$\mathrm{UNIT}(\underline{Z}_{MF})=\mathrm{UNIT}(\underline{R}_{S/C} \times \underline{Y}_{MF})$$

$$\left.\begin{array}{l}\text{XSE, YSE, ZSE: SPACECRAFT POSITION ERROR}\\ \text{XSDE, YSDE, ZSDE: SPACECRAFT VELOCITY ERROR}\\ \\ \text{XTE, YTE, ZTE: TARGET POSITION ERROR}\\ \text{XTDE, YTDE, ZTDE: TARGET VELOCITY ERROR}\end{array}\right\} \text{STATE ERRORS IN MF}$$

Relative State Errors: Bias estimation portion is the estimated bias minus actual bias for each sensor in ft, fps, mr.

XRE, YRE, ZRE: RELATIVE POSITION ERRORS IN MF (ft)
XRDE, YRDE, ZRDE: RELATIVE VELOCITY ERRORS IN MF (fps)

Relative Parameters: Navigated and actual values of range, range-rate, azimuth and elevation in the local vertical in-plane frame. Measured values of these quantities have noise and biases added and are given in the estimated measurement frame. Units are ft, fps, deg.

Area C: Navigation Status

MFLG: Filter status flag: MFLG=0 Marking is enabled on any procedure active during this period.
MFLG=1 Marking is suspended because of final maneuver computation or recycle
MFLG=2 Final comp has been done and the filter is waiting for ignition before resuming updates

RAD: Number of radar marks since last W-reinitialization

VHF: Number of VHF marks since last W-reinitialization

OPT: Number of angle marks since last W-reinitialization

UR: State uncertainty in next range measurement (ft)

URD: State uncertainty in next r-rate measurement (fps)

UAZ: State uncertainty in next azimuth measurement (mr)

UEL: State uncertainty in next elevation measurement (mr)

SR: Weighting given a-priori estimate of range on last mark (A value of 1.0 indicates 100% confidence)

SRD: Same as above for r-rate

SAZ: Same as above for azimuth

SEL: Same as above for elevation

Area D: Coelliptical relative errors at phase match. Space craft is advanced to phase match with target and curvilinear errors are computed

S-R: Down-range error

S-DOT: Horizontal speed error

SD-COEL: Horizontal speed error minus speed error if vehicle errors were coelliptical

DELTA-H: Vertical position error

DELH-DOT: Vertical speed error

Area E: List of square roots of the diagonal terms of the filter covariance matrix. 1-sigma estimated uncertainty in the value of each estimated quantity, expressed in measurement frame. Vehicle rows are in the order of X, Y, Z, XD, YD, ZD; bias row is in order of R, RDOT, AZ, EL, SC1,SC2.

Area F: Covariance of relative errors in the measurement frame. Diagonal elements are 1-sigma uncertainties in each of the frame directions in the order of X, Y, Z, XD, YD, ZD. Lower left portion (Area F1) is array of correlation coefficients.

Area G: Estimated and actual maneuver. The navigation burn estimate is that computed from the estimated vehicle states. The actual value is that maneuver actually applied in view of platform drift, scale factor error and cutoff uncertainty.

MASS DATA STORAGE

On each cycle of a monte-carlo set, AAP stores up to 350 items of data on a locally created file, in unformatted form, for later transference to a permanent data storage tape. Before writing the local file, the program writes the date, time and number of cycles (one word each) on the beginning of the file. At the end of each cycle, the DATA array of 350 words is dumped on the file. These words are allotted as follows:

DATA(1)=LOOP, Current value of the monte-carlo cycle index

DATA(2)-DATA(249), Up to 8 sets of state vector and maneuver information stored for selected maneuvers (see CARD #4). Each set consists of 31 elements:
1-6: S/C actual BRF vector
7-12: TGT actual BRF vector
13-18: S/C navigated BRF vector
19-24: TGT navigated BRF vector
25-27: Maneuver computed from actual states
28-30: Maneuver computed from estimated states
31: Time of ignition

DATA(251)-DATA(320), Up to ten sets of platform alignment information. Each set consists of 7 elements:
1-3: Platform angular drift rates (X,Y,Z)
4-6: Initial platform misalignment (X,Y,Z)
7: Time of alignment

DATA(321)-DATA(350), Arbitrary output specified by user.

Regardless of which maneuver is the first with a data storage flag set, the state vector/maneuver data is stacked, 31 elements at a time, beginning in DATA(2). All quantities are output in fundamental units of feet, seconds and radians. A dump of the DATA array in the indicated format is performed at the termination of each cycle.

5.0 PROGRAM MECHANIZATION

This section will discuss the implementation of the navigation function down to the level of FORTRAN code. As a preliminary, the assignment and definition of all variables associated with the navigation will be reviewed.

MASTER COMMON AND EQUIVALENCE LIST

BLANK COMMON:	VAR(5600)	BETELGEUSE BLANK COMMON
LABEL COMMON:	DELV	LISTS VARS, VARA, NFAMA, SD(3) FOR USE IN MANEUVER APPLICATION ROUTINE. VARS, VARA, AND NFAMA ARE DISCUSSED IN SECTION 3.1. SD(3) ARE THREE ACCELEROMETER SCALE FACTORS CREATED ON FIRST VISIT TO DELTAV (MANEUVER APPLICATION).
	GARB	LISTS ACTUAL STATISTICS OF SENSOR ERRORS FOR USE BY SENSOR NOISE ROUTINE (GARBAGE). SEE SECTION 3.1.
	ALIG	LISTS ACTUAL STATISTICS OF PLATFORM ALIGNMENT AND DRIFT ERRORS FOR USE BY PLAT ALIGN ROUTINE (ALIGN). SEE SECTION 3.1.
	BV	LISTS FILTER STATISTICS OF SENSOR ERRORS FOR USE BY GEOMETRY VECTOR SUBROUTINE (BVEC). SEE SECTION 3.1.

DIMENSIONED ARRAYS USED BY NAVIGATION

Y(100)	INTEGRATED VARIABLES
DYDX(100)	DERIVATIVES WRT TIME
NTEGER(100)	INTEGER PARAMETERS
P(5000)	BETELGEUSE WORKING ARRAY
SAVE(950)	NAVIGATION STORAGE ARRAY
BLK(700)	NAVIGATION SCRATCH PAD
DATA(350)	MONTE-CARLO MASS DATA STORAGE
COV(24,24)	INPUT STATE VECTOR COVARIANCE MATRIX
QQ(4)	MEASURED RELATIVE PARAMETERS
SIG(4)	CURRENT VALUE OF MEASURED RELATIVE PAR RANDOM NOISE
C(10)	ARRAY OF CONTROL TIMES FOR GNEXEC
REFMAT(3,3)	ESTIMATED TRANSFORMATION MATRIX FROM BRF TO PLATFORM AXES
XNBN(3), YNBN(3), ZNBN(3)	ESTIMATED NAVIGATION BASE UNIT VECTORS
NE(10)	SEE SECTION 3.2
NM(10)	" " "
DTL(10)	" " "
DTN(10)	" " "
DTM(10)	" " "
USP(10)	" " "
USV(10)	" " "
UTP(10)	" " "
UTV(10)	" " "

```
OWING AREA IS BLANK COMMON FOR BETELGEUSE                                     POPOUT      6
    COMMON VAR                                                                POPOUT      7
                                                                              POPOUT      8
+
 MF  AT, MFTRN                                                                POPOUT      9
+                                                                             POPOUT     10
NSIO  VAR(5600), Y(100), DYDX(100), Q(100), FIRSTY(100)                       POPOUT     11
     NTEGER(100), D(100), P(5000)                                             POPOUT     12
VALENCE (VAR(1),Y(1))                                                         POPOUT     13
        (VAR(101),DYDX(1))                                                    POPOUT     14
        (VAR(201),Q(1))                                                       POPOUT     15
        (VAR(301),FIRSTY(1))                                                  POPOUT     16
        (VAR(401),NTEGER(1))                                                  POPOUT     17
        (VAR(501),D(1))                                                       POPOUT     18
        (VAR(601),P(1))                                                       POPOUT     19
NSION SAVE(950), BLK(700), DATA(350), COV(24,24)                              POPOUT     20
VALENCE (P(350),SAVE(1))                                                      POPOUT     21
        (P(1300),BLK(1))                                                      POPOUT     22
        (P(4074),DATA(1))                                                     POPOUT     23
        (P(4424),COV(1,1))                                                    POPOUT     24
NSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)                     POPOUT     25
      ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)                      POPOUT     26
      USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)                     POPOUT     27
      SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)                        POPOUT     28
      ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)                    POPOUT     29
      X(18), WE(18,27)                                                        POPOUT     30
      XLVE(3),YLVE(3),ZLVE(3),XLVN(3),YLVN(3),ZLVN(3)                         NOSHIT      1
VALENCE (SAVE(1),QQ(1)),               (SAVE(5),SIG(1))                       POPOUT     31
        (SAVE(9),C(1)),                (SAVE(19),REFMAT(1,1))                 POPOUT     32
        (SAVE(28),XNBN(1)),            (SAVE(31),YNBN(1))                     POPOUT     33
        (SAVE(34),ZNBN(1)),            (SAVE(37),NE(1))                       POPOUT     34
        (SAVE(47),NM(1)),              (SAVE(57),DTL(1))                      POPOUT     35
        (SAVE(67),DTN(1)),             (SAVE(77),DTM(1))                      POPOUT     36
        (SAVE(87),USP(1)),             (SAVE(97),USV(1))                      POPOUT     37
        (SAVE(107),UTP(1)),            (SAVE(117),UTV(1))                     POPOUT     38
        (SAVE(127),SR(1)),             (SAVE(137),SRD(1))                     POPOUT     39
        (SAVE(147),SO(1)),             (SAVE(157),SC1(1))                     POPOUT     40
        (SAVE(167),SC2(1)),            (SAVE(177),NW(1))                      POPOUT     41
        (SAVE(187),TLM(1)),            (SAVE(197),NS(1))                      POPOUT     42
        (SAVE(200),ZTZ(1)),            (SAVE(204),SZ(1))                      POPOUT     43
        (SAVE(208),TALIGN(1)),         (SAVE(218),NALIGN)                     POPOUT     44
        (SAVE(229),XNBE(1)),           (SAVE(232),YNBE(1))                    POPOUT     45
        (SAVE(235),ZNBE(1)),           (SAVE(258),X(1))                       POPOUT     46
        (SAVE(276),WE(1,1))                                                   POPOUT     47
        (NTEGER(30),ICOMP)                                                    POPOUT     48
VALENCE (SAVE(762),XLVE(1))                                                   NOSHIT      2
       (SAVE(765),YLVE(1))                                                    NOSHIT      3
       (SAVE(768),ZLVE(1))                                                    NOSHIT      4
       (SAVE(771),XLVN(1))                                                    NOSHIT      5
       (SAVE(774),YLVN(1))                                                    NOSHIT      6
       (SAVE(777),ZLVN(1))                                                    NOSHIT      7
                                                                              POPOUT     49
```

FIGURE 5.1 STANDARD COMMON BLOCK

```
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```

SR(10)	SEE SECTION 3.2
SRD(10)	" " "
SO(10)	" " "
SC1(10),SC2(10)	" " "
NW(10)	" " "
TLM(10)	STORED TIME OF THE LAST MARK ON EACH NAVIGATION PROCEDURE
NS(3)	NUMBER OF MARKS SINCE W-REINITIALIZATION ON RADAR, VHF AND OPTICS
ZTZ(4)	A PRIORI UNCERTAINTY IN VALUE OF SENSOR MEASUREMENT
SZ(4)	WEIGHTING ON A PRIORI ESTIMATE OF SENSOR MEASUREMENT
TALIGN(10)	STORED TIMES OF PLATFORM ALIGNMENTS SEE SECTION 3.2
XNBE(3),YNBE(3), ZNBE(3)	ACTUAL NAVIGATION BASE UNIT VECTORS
X(18)	LOCAL STORAGE FOR ESTIMATED CARTESIAN STATE
WE(18,18)	THE FAMOUS W-MATRIX
XLVE(3),YLVE(3) ZLVE(3)	ACTUAL LOCAL VERTICAL UNIT VECTORS
XLVN(3),YLVN(3) ZLVN(3)	ESTIMATED LOCAL VERTICAL UNIT VECTORS
DRIFT(3)	INTEGRATED PLATFORM MISALIGNMENT ANGLES
RATE(3)	PLATFORM GYRO DRIFT RATES

Subroutines which utilize the BLK array for intermediate local computations define and equivalence local variables as required. These will be individually discussed in the section on subroutine structure. Figure 5.1 presents the standard navigation common block . In addition to the equivalences shown, the following are scattered throughout:

NTEGER(29),	NGUIDE	CURRENT VALUE OF NGUIDE
30	ICOMP	NAVIGATION STATUS FLAG (SEE MFLG, SECTION 4.0, AREA C)
31	ISTEP	GNEXEC INTEGRATION STEP-SIZE MANAGEMENT FLAG
32	NOVER	OVERLAY RETURN MANAGEMENT FLAG
33	LIGN	CURRENT NUMBER OF PLATFORM ALIGNMENTS ALREADY PERFORMED
34		NOT DEFINED
35	NGATE	BRAKING GATE COUNTER
36	NBRFL	SET WHEN IN BRAKING PHASE
42	NLINE	PRINTED OUTPUT LINE COUNTER
C(1)	TW	TIME TAG ON W-MATRIX
8	STEP	SAVED VALUE OF INPUT INTEGRATION STEP SIZE
9	T2	SYNCH STEP SIZE FOR FINAL PASS AFTER MANEUVER APPLICATION
10	TGN	TIME FROM NEXT IGNITION

5.1 FUNCTIONAL DESCRIPTION

In spite of the overlay structure, the actual operation of AAP is practically indestinguishable from that of the traditional BETELGEUSE program. The guidance overlay (2nd) acts like a subroutine which when called, computes a maneuver specified by the current value of NGUIDE. Other than the filter computation routines, the most significant addition to the program is the guidance and navigation executive, which controls the taking of navigation marks and the calling of the guidance overlay. The following is a list of subroutines peculiar to or modified by the presence of the navigation function:

MAIN	REFERENCES MONTE-CARLO MASS STORAGE FILE (TAPE77=STAT) SETS VALUES FOR ERROR MODELS BY REPLACEMENT
RK	HAS OVERLAY RETURN FLAG (NOVER), CALL TO GNEXEC, CALL TO SETY, CALL TO POPOUT, ENTRY POINT FOR COVARIANCE ADVANCEMENT (ENTRY RKW), INTEGRATES PLATFORM DRIFTS
INPUT	WRITES ON MASS DATA STORAGE TAPE (77), READS DATA STORAGE CONTROL FLAGS, READS NAVIGATION PROCESS CONTROL CARDS, READS ALIGNMENT CONTROL CARD, INITIALIZES GUIDANCE, NAVIGATION AND ALIGNMENT PARAMETERS, CALLS ALIGN AND SETY
GNEXEC	CONTROLS TAKING OF NAVIGATION MARKS, COMPUTATION AND APPLICATION OF MANEUVERS
DELTAV	APPLIES AN ESTIMATED AND ACTUAL MANEUVER TO THE ESTIMATED AND ACTUAL STATES
STORE1	STORES 31 ELEMENTS OF STATE VECTOR AND MANEUVER INFORMATION AT SELECTED MANEUVERS
OUTDAT	DUMPS THE MASS STORAGE MONTE-CARLO DATA AT THE END OF EACH CYCLE
POPOUT	COMPUTES ACTUAL, ESTIMATED AND MEASURED RELATIVE PARAMETERS, SETS UP VECTORS AND FILTER STATUS DATA, PRINTS BLOCK DATA AS REQUIRED
CART1,CART2	SUBROUTINE CONVERTS BETELGEUSE VECTOR TO CARTESIAN (CART1), OR A CARTESIAN VECTOR TO BETELGEUSE (CART2)
SETY	LOADS CARTESIAN FORM OF ESTIMATED BETELGEUSE STATE INTO Y ARRAY. CURRENTLY ONLY ONE INSTRUCTION IS ACTIVE- ALL CALLS TO SETY MAY BE REPLACED WITH THE STATEMENT CALL CART1(Y(38),Y(2))
REL	COMPUTES RELATIVE PARAMETERS (R, RDOT, AZ, EL) GIVEN CARTESIAN VECTOR AND UNIT VECTORS OF MEASUREMENT FRAME

SETUP	SETS UP AN OUTPUT FORM OF THE BETELGEUSE VECTOR
GARBAGE	CREATES INITIAL SENSOR BIASES AND ADDS RANDOM NOISE TO RELATIVE PARAMETERS
ALIGN	CREATES INITIAL PLATFORM DRIFT RATES, DEFINES A REFMAT AND SETS MISALIGNMENT BIASES FOR EACH ALIGNMENT
P20	LOCAL SUPERVISORY ROUTINE FOR TAKING A NAVIGATION MARK
ADVW	COVARIANCE INTEGRATING SUBROUTINE
BVEC	CALCULATES AND CONSTRUCTS REQUIRED GEOMETRY VECTORS FOR FILTER
FILTER	CALCULATES WEIGHTING VECTOR, UPDATES STATE AND COVARIANCE
OPEN1-OPEN5	DUMMY SUBROUTINES FOR USER DEFINED FUNCTIONS

The navigation function operates on a cartesian vector which is created periodically from the estmated BETELGEUSE state vector. This cartesian vector, although not itself integrated, is stored in the Y array. Because it is required for covariance advancement, the previous value of this vector , called X, is stored in the SAVE array from the previous visit to ADVW. The allocation of the Y array to state and other variables is as follows:

Y(1)	TIME
Y(2)-Y(13)	ESTIMATED BETELGEUSE STATE
Y(14)-Y(19)	ESTIMATED VALUES OF SENSOR BIASES
Y(20)-Y(31)	ACTUAL BETELGEUSE STATE
Y(32)-Y(37)	ACTUAL VALUE OF SENSOR BIASES
Y(38)-Y(49)	ESTIMATED CARTESIAN STATE
Y(50)-Y(55)	ESTIMATED VALUE OF SENSOR BIASES (SAME AS Y(14)-Y(19))
Y(98)-Y(100)	TOTAL INTEGRATED PLATFORM DRIFT ANGLES. THESE ARE EQUIVALENCED TO DRIFT(3) IN SUBROUTINE ALIGN. ALSO, THEIR DERIVATIVES, RATE(3), ARE EQUIVALENCED TO DYDX(98)-DYDX(100) IN ALIGN.

5.2 COMPUTATIONAL ORGANIZATION

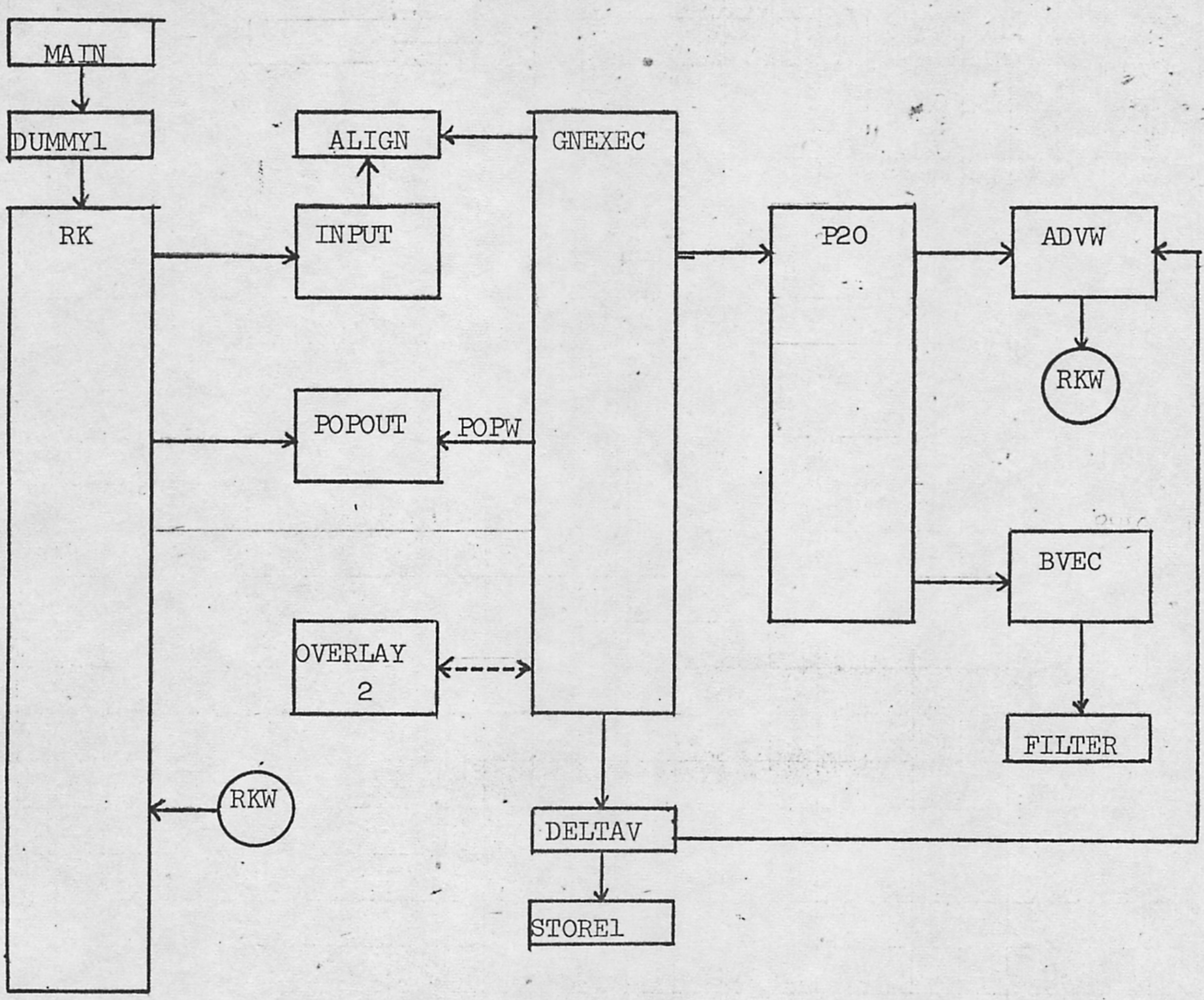
Figure 5.2.1 presents the interfaces of the major navigation functional subroutines. The remaining part of this section will present the FORTRAN code used to implement this function, with flow diagrams and explainatory notes where apprpriate. The following comments refer to Figure 5.2.1:

MAIN	Main program of main overlay.
DUMMY1	Called by MAIN to bring in first primary overlay AAP(1,0)
RK	Integrating subroutine for (1,0). Also called at ENTRY RKW by ADVW for integration of W-matrix.
INPUT	Controls input of data to program. Called by RK at the beginning of each monte-carlo cycle
ALIGN	Simulates performance of platform alignment. Called by INPUT at beginning of program execution, and by GNEXEC at times defined by alignment control card.
POPOUT	Handles computation of actual, estimated and measured relative quantities for output and navigation. Computes and organizes for output other quantities of interest. Prints block data at intervals defined by P(9), and whenever called through ENTRY POPW by GNEXEC. Computation section of POPOUT will be executed when POPOUT is called on each pass through RK, even if print is inhibited.
OVERLAY 2	Called in from GNEXEC at termination of the last marking procedure in a premaneuver period. Because return from AAP(2,0) is to DUMMY1 and RK, special provision is made in the program to return to the next instruction in GNEXEC following the call to AAP(2,0). This is accomplished by the setting of the NOVER flag.
GNEXEC	Executive routine. Controls taking of navigation marks, performance of platform alignments and computation of maneuvers. Called once each pass through RK.
DELTAV	Applies maneuvers computed by guidance overlay to actual and estimated states. Called from GNEXEC. Also calls ADVW to advance covariance to time of ignition.
STORE1	Loads state vector and delta-v data into DATA array for dump to mass storage at end of cycle. Called from GNEXEC.
P20	Controls advancement of W, taking mark, updating state

ADVW — ADVANCES W-MATRIX TO CURRENT TIME WHENEVER CALLED. ENTERS RK AT RKW FOR COLUMEN ADVANCEMENT OF MATRIX.

BVEC — DETERMINES WHAT SORT OF MARK IS DESIRED, CALCULATES APPROPRIATE GEOMETRY VECTORS, CALLS FILTER TO UPDATE STATE AND COVARIANCE

FILTER — CALCULATES WEIGHTING VECTOR, UPDATES STATE AND COVARIANCE.



NOTE: ARROWS GO FROM CALLING ROUTINE TO CALLED ROUTINE.

FIGURE 5.2.1 NAVIGATION INTERFACES

MAIN OVERLAY

```
      OVERLAY(AAP,0,0)
      PROGRAM MAIN(INPUT,OUTPUT,STAT,TAPE5=INPUT,TAPE6=OUTPUT
     *,                                  TAPE77=STAT)
C     GENERAL SUBROUTINES FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS
C     BY MEANS OF FOUR POINT RUNGE KUTTA NUMERICAL INTEGRATION
C
            COMMON VAR
C
C     FOLLOWING IS LABELED COMMON FOR ERROR MODELS AND NAVIGATION
C
      COMMON/DELV/VARS,VARA,NFAMA,SD(3)
C
      COMMON/GARB/RVAR,RVARMIN,VVAR,VVARMIN,VARAZ,VAREL,NFAMV
     *,         BR,BV,BAZ,BEL,NFAMB
     *,         BRO,BVO,BAZO,BELO
C
      COMMON/ALIG/GDR,ALIGNB,NFAMC
C
      COMMON/BV/RVARB,RVARMNB,VVARB,VVARMNB,VARAZB,VARELB
      DIMENSION SAVE(950), BLK(700)
      EQUIVALENCE (P(350),SAVE(1))
     *,         (P(1300),BLK(1))
C
            DIMENSION Y(100), DYDX(100), Q(100), FIRSTY(100),
     1 P(5000),NTEGER(100),VAR(5600),D(100)
            EQUIVALENCE (VAR(1),Y(1)), (VAR(101),DYDX(1)),
     1 (VAR(201),Q(1)), (VAR(301),FIRSTY(1)), (VAR(401),NTEGER(1)),
     2 (VAR(501),D(1)), (VAR(601),P(1))
            EQUIVALENCE   (P(2301),TWOPI), (P(2302),CRAD),
     1 (P(2303),CNM)
```

```
C     ZERO CORE AT INITIAL LOADING                                      MAIN      33
          DO 20 J=1,5600                                                MAIN      34
   20     VAR(J) = 0.0                                                  MAIN      35
C     SET IN FUNDAMENTAL CONSTANTS                                      MAIN      36
          TWOPI=6.2831853072                                            MAIN      37
          CRAD=57.2957795131                                            MAIN      38
          CNM=6076.10333                                                MAIN      39
C     LOAD ERROR MODEL                                                  MAIN      40
      VARS=1.E-4                                                        MAIN      41
      VARA=1.E-1                                                        MAIN      42
C                                                                       MAIN      44
        RVAR=0.                                                         SENSOR     1
        RVARMIN = 33.                                                   SENSOR     2
      VVAR=4.3E-3                                                       MAIN      47
      VVARMIN=.43                                                       MAIN      48
        VARAZ=2.E-3                                                     SENSOR     3
        VAREL=2.E-3                                                     SENSOR     4
      BR=0.                                                             MAIN      52
      BV=0.                                                             MAIN      53
      BAZ=0.                                                            MAIN      54
      BEL=0.                                                            MAIN      55
      BRO=0.                                                            MAIN      57
      BVO=0.                                                            MAIN      58
      BAZO=17.45E-3                                                     MAIN      59
```

PROGRAM MAIN CDC 6600 FTN V3.0-P308 OPT=1 08/29/72 11.32.27.

```
      BELO=17.45E-3                                                     MAIN      60
C                                                                       MAIN      61
      GDR=1.45E-7                                                       MAIN      62
      ALIGNB=3.E-4                                                      MAIN      63
      NFAMA= 46728                                                      RANNO      1
      NFAMB= 12944                                                      RANNO      2
      NFAMC= 31171                                                      RANNO      3
      NFAMV= 22222                                                      RANNO      4
C                                                                       MAIN      65
        RVARB=0.                                                        SENSOR     5
        RVARMNB = 33.                                                   SENSOR     6
      VVARB=4.3E-3                                                      MAIN      68
```

```
      VARAZB=2.E-3                                                      SENSOR
      VARELB=2.E-3                                          A           SENSOR
                                                                        MAIN
C     SET DERIVATIVE OF INDEPENDENT VARIABLE WR/T ITSELF EQUAL TO ONE   MAIN
   30       DYDX(1) = 1.0                                               MAIN
C     SET DERIV OF BIAS TERMS EQUAL ZERO                    B           MAIN
            DO 35 I=1,6                                                 MAIN
            DYDX(I+13)=0.                                               MAIN
            DYDX(I+31)=0.                                               MAIN
   35       CONTINUE                                        B           MAIN
C     SET JOB COUNTER                                                   MAIN
            LOOP=0                                                      MAIN
C     TRANSFER CONTROL TO NAVIGATION OVERLAY                            MAIN
            CALL DUMMY1                                                 MAIN
            END                                                         MAIN
```

```
          SUBROUTINE DUMMY1                                    MAIN      85
C     ROUTINE TO LOAD NAVIGATION OVERLAY                       MAIN      86
          CALL OVERLAY(3HAAP,1,0,6HRECALL)                     MAIN      87
          RETURN                                               MAIN      88
          END                                                  MAIN      89
```

```
      SUBROUTINE DUMMY2                                   MAIN      90
C     ROUTINE TO LOAD GUIDANCE OVERLAY                    MAIN      91
      CALL OVERLAY(3HAAP,2,0,6HRECALL)                    MAIN      92
      RETURN                                              MAIN      93
      END                                                 MAIN      94
```

1st Primary RK, DYDXS

```
      OVERLAY(1,0)                                               MAIN    95
      PROGRAM NAVLAY                                             MAIN    96
C     TRANSFER CONTROL TO INTEGRATION SUBROUTINE                 MAIN    97
      CALL RK                                                    MAIN    98
      END                                                        MAIN    99
```

PROGRAM MAIN

AREA A THE PURPOSE AND HANDLING OF THESE INSTRUCTIONS IS DISCUSSED IN SECTION 3.1

AREA B TIME DERIVATIVES OF BIAS PORTION OF INTEGRATED STATE VECTORS IS SET EQUAL TO ZERO. ALL ESTIMATED SENSOR BIASES ARE ASSUMED CONSTANT.

ROUTINE RK

AREA A IF THIS PASS THROUGH RK IS A RETURN FROM THE GUIDANCE OVERLAY, NOVER WILL BE SET TO 1,2,3 OR 4, DEPENDING ON THE LOCATION IN GNEXEC WHICH CALLED THE OVERLAY. IN THIS CASE, IT IS DESIRED TO GO DIRECTLY BACK TO GNEXEC.

AREA B CALL TO SETY LOADS A CARTESIAN FORM OF THE BETELGEUSE ESTIMATED STATE INTO Y(38) - Y(55) FOR USE BY OTHER SUBROUTINES. THIS MUST BE ACCOMPLISHED EACH INTEGRATION STEP BEFORE VISITING OUTPUT OR NAVIGATION EXECUTIVE.

AREA C CALL TO POPOUT ACCOMPLISHES COMPUTATION OF RELATIVE STATE QUANTITIES FOR USE BY NAVIGATION, AND PERIODIC PRINTING OF BLOCK DATA DESCRIBED IN SECTION 4.0

AREA D GUIDANCE AND NAVIGATION IS VISITED EACH INTEGRATION CYCLE TO PROVIDE FOR PERFORMANCE OF NAVIGATION, COMPUTATION OF MANEUVERS AND MANEUVER APPLICATION

AREA E RK WILL BE CALLED PERIODICALLY AT ENTRY RKW FROM ADVW FOR ADVANCEMENT OF COVARIANCE. IN THIS CASE, IT IS DESIRED TO GO TO THE RETURN STATEMENT AT THE CONCLUSION OF THE STATE INTEGRATION INSTRUCTIONS. BY SETTING NFLGW=1 UPON ENTRY AT RKW, PROGRAM WILL RETURN TO CALLING ROUTINE ADVW INSTEAD OF PROCEEDING TO NEXT INTEGRATION STEP. FLAG IS RESET UPON NEXT NORMAL PASS THROUGH RK.

AREA F LOGICAL OPERATOR TRANSFERS CONTROL TO RETURN STATEMENT IF NFLGW IS SET. OTHERWISE PLATFORM DRIFTS ARE INTEGRATED ONE STEP BEFORE GOING ON TO NEXT PASS THROUGH RK.

```
            SUBROUTINE RK
C     GENERAL SUBROUTINES FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS
C     BY MEANS OF FOUR POINT RUNGE KUTTA NUMERICAL INTEGRATION
C
C     RK - INTEGRATING SUBPROGRAM
            COMMON VAR
            DIMENSION Y(100), DYDX(100), Q(100), FIRSTY(100),
    1 P(5000),NTEGER(100),VAR(5600),NT1(14),NT2(14),NT(14),D(100)
            EQUIVALENCE (VAR(1),Y(1)), (VAR(101),DYDX(1)),
    1 (VAR(201),Q(1)), (VAR(301),FIRSTY(1)), (VAR(401),NTEGER(1)),
    2 (VAR(501),D(1)), (VAR(601),P(1)), (NTEGER(6), N)
    3,             (NTEGER(32),NOVER)
C     CHECK IF THIS IS A RETURN FROM AN OVERLAY CALL          A
      IF(NOVER.GT.0) GO TO 25                                  A
C     LOAD INPUT DATA INTO COMPUTER
   10       CALL INPUT
C     CALCULATE THE INITIAL VALUES OF THE DERIVATIVES
   20       CONTINUE
      CALL SETY(Y)                                             B
      CALL POPOUT                                              C
   25 CONTINUE
      CALL GNEXEC                                              D
            IF(Y(1).GT.P(2)) GO TO 10
      NFLGW = 0                                                E
      GO TO 29
      ENTRY RKW
      NFLGW = 1                                                E
   29 CONTINUE
            CALL DYDXS
C     WRITE INITIAL VALUES OF DERIVATIVES
   30       CONTINUE
C     CALCULATE THE DELTA Y(J) AT Y(1) = 0
   40       DO 50 J = 1,N
   50       D(J) = DYDX(J)*P(1)
C     CALCULATE THE Y(J) AT T = 0
   60       DO 90 J = 1,N
   70       R = .5*(D(J) - Q(J))
   80       Y(J) = Y(J) + R
   90       Q(J) = Q(J) + 3.0*R - .5*D(J)
C     CALCULATE THE DELTA Y(J) AT Y(1) = HALF STEP
  100       CALL DYDXS
  110       DO 120 J = 1,N
```

```
C         CALCULATE THE Y(J) AT Y(1) = HALF STEP
  130           DO 160  J = 1,N
  140           R = .292893219*(D(J) - Q(J))
  150           Y(J) = Y(J) + R
  160           Q(J) = Q(J) + 3.0*R - .292893219*D(J)
C         CALCULATE THE DELTA Y(J) AT Y(1) = HALF STEP (AGAIN)
  170           CALL DYDXS
  180           DO 190  J = 1,N
  190           D(J) = DYDX(J)*P(1)
C         CALCULATE THE Y(J) AT Y(1) = HALF STEP (AGAIN)
  200           DO 230  J = 1,N
  210           R = 1.70710678*(D(J) - Q(J))
```



```
  220           Y(J) = Y(J) + R
  230           Q(J) = Q(J) + 3.0*R - 1.70710678*D(J)
C         CALCULATE THE DELTA Y(J) AT Y(1) = FULL STEP
  240           CALL DYDXS
  250           DO 260  J = 1,N
  260           D(J) = DYDX(J)*P(1)
C         CALCULATE THE Y(J) AT Y(1) = FULL STEP
  270           DO 300  J = 1,N
  280           R = .1666666667*(D(J) - 2.0*Q(J))
  290           Y(J) = Y(J) + R
  300           Q(J) = Q(J) + 3.0*R - .5*D(J)
          IF(NFLGW.EQ.1) GO TO 330                                    F
C         INTEGRATE PLATFORM DRIFTS
                DO 305 I=98,100
  305           Y(I)=Y(I) + P(1)*DYDX(I)                              F
C         PROCEED TO THE NEXT INTEGRATION STEP
  310           NGO = 1
  320           GO TO (20,330),NGO
  330           RETURN
                END
```

INPUT

ROUTINE INPUT

AREA A — FIRST TWO INSTRUCTIONS CHECK NW(I) ARRAY (SEE SEC 3.2) TO RESET W-MATRIX REINITIALIZATION FLAGS. IF ANY FLAG OF THIS ARRAY IS 0 ON INPUT NAVIGATION CONTROL CARD, IT IS SET TO -1 AT TIME W IS REINITIALIZED. LAST FOUR INSTRUCTIONS IN THIS ARE WRITE OUT ACCUMULATED MONTE-CARLO DATA TO LOCAL MASS STORAGE FILE AND THEN ZERO ARRAY FOR NEXT CYCLE.

AREA B — AT BEGINNING OF PROGRAM EXECUTION, SYSTEM ROUTINES ARE READ TO DETERMINE DATE AND TIME OF THIS RUN. THIS INFORMATION PLUS THE NUMBER OF MONTE-CARLO CYCLES TO BE EXECUTED ARE THE READ ONTO BEGINNING OF MASS STORAGE FILE.

AREA C — FOLLOWING INPUT OF BETELGEUSE HOLLERITH COMMENT CARDS, A CARD SPECIFYING THE MANEUVERS AT WHICH DATA WILL BE STORED IS READ IN AND PRINTED OUT. NO MORE THAN 8 ELEMENTS OF NST(I) MAY BE NON-ZERO; THE DATA ARRAY HAS ROOM FOR ONLY 8 MANEUVERS WORTH OF DATA.

AREA D — AFTER INPUT OF BETELGEUSE FLOATING POINT ENTRIES, A SERIES OF UP TO 10 NAVIGATION CONTROL CARDS IS READ IN. NUMBER OF THESE CARDS IS EQUAL TO NPER. AFTER INPUT, TIME, LENGTH, AND ANGLE QUANTITIES ARE RESCALED TO FUNDAMENTAL UNITS OF SECONDS, FEET AND RADIANS.

AREA E — FOLLOWING NAVIGATION CONTROL CARDS, AN ALIGNMENT CONTROL CARD, AS SPECIFIED IN SEC 3.2 IS READ AND ALIGNMENT TIMES PRINTED OUT.

AREA F — DATA ARRAY STACK INDEX IS RESET. THIS INDEX IS INCREMENTED BY 1 EACH TIME A 31 ELEMENT ARRAY OF VECTOR AND MANEUVER DATA IS STORED IN THE DATA ARRAY. SEE MASS DATA STORAGE, PAGE 9.

AREA G — RESET MANEUVER TYPE COMPUTATION FLAG TO STARTING VALUE.
RESET ALIGNMENT COUNTER FLAG (INCREMENTED BY 1 EACH TIME AN ALIGNMENT IS PERFORMED.
SET MANEUVER COMPUTATION FLAG TO CALL FOR AN INITIAL VISIT TO THE GUIDANCE OVERLAY AT FIRST VISIT TO GNEXEC.

AREA H — ZERO THE W-MATRIX TO BE SURE ITS NICE AND CLEAN FOR NEXT CYCLE.
LOAD ESTIMATED CARTESIAN STATE INTO Y(38)-Y(55).
PERFORM INITIAL ALIGNMENT TO DEFINE PLATFORM AXES.

```
          SUBROUTINE INPUT
C     GENERAL SUBROUTINES FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS
C     BY MEANS OF FOUR POINT RUNGE KUTTA NUMERICAL INTEGRATION
C
C     INPUT - SUBPROGRAM FOR READING IN DATA
          COMMON VAR
          DIMENSION Y(100), DYDX(100), Q(100), FIRSTY(100),
     1 P(5000),NTEGER(100),VAR(5600),NT1(14),NT2(14),NT(14),D(100)
          EQUIVALENCE (VAR(1),Y(1)), (VAR(101),DYDX(1)),
     1 (VAR(201),Q(1)), (VAR(301),FIRSTY(1)), (VAR(401),NTEGER(1)),
     2 (VAR(501),D(1)), (VAR(601),P(1)), (NTEGER(6),N),
     3 (NTEGER(1),IDENT), (NTEGER(2),NP), (NTEGER(3),NINT),
     4 (NTEGER(4),NFIRST), (NTEGER(5),NTABLE), (NTEGER(7),NMORE),
     5 (NTEGER(8),NT(1)), (NTEGER(23),NTCR), (NTEGER(24),NTSKIP),
     6 (NTEGER(41),LPRINT), (NTEGER(42),NLINE), (NTEGER(43),NSKIP),
     7 (NTEGER(44),NPAGE), (NTEGER(45),NT1(1)), (NTEGER(60),NT2(1))
      DIMENSION NST(15), TIG(15)
      EQUIVALENCE (P(294),NFAM2)
     *,          (P(295),LOOP)
     *,          (P(334),NSTAT)
     *,          (P(335),NST(1))
     *,          (P(2107),TTPI)
     *,          (P(2141),TIG(1))
      EQUIVALENCE (NTEGER(12),NPER)
     *,          (NTEGER(29),NGUIDE)
     *,          (NTEGER(30),ICOMP)
     *,          (NTEGER(33),LIGN)
     *,          (NTEGER(35),NGATE)
     *,          (NTEGER(36),NBRFL)
      DIMENSION SAVE(950), BLK(700), DATA(350), COV(24,24)
      EQUIVALENCE (P(350),SAVE(1))
     *,          (P(1300),BLK(1))
     *,          (P(4074),DATA(1))
     *,          (P(4424),COV(1,1))
      DIMENSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)
     *,        ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)
     *,        USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)
     *,        SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)
     *,        ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)
     *,        X(18), WE(18,27)
      EQUIVALENCE (SAVE(1),QQ(1)),          (SAVE(5),SIG(1))
     *,          (SAVE(9),C(1)),            (SAVE(19),REFMAT(1,1))
```

```
     *,           (SAVE(34),ZNBN(1)),       (SAVE(37),NE(1))
     *,           (SAVE(47),NM(1)),         (SAVE(57),DTL(1))
     *,           (SAVE(67),DTN(1)),        (SAVE(77),DTM(1))
     *,           (SAVE(87),USP(1)),        (SAVE(97),USV(1))
     *,           (SAVE(107),UTP(1)),       (SAVE(117),UTV(1))
     *,           (SAVE(127),SR(1)),        (SAVE(137),SRD(1))
     *,           (SAVE(147),SO(1)),        (SAVE(157),SC1(1))
     *,           (SAVE(167),SC2(1)),       (SAVE(177),NW(1))
     *,           (SAVE(187),TLM(1)),       (SAVE(197),NS(1))
     *,           (SAVE(200),ZTZ(1)),       (SAVE(204),SZ(1))
     *,           (SAVE(208),TALIGN(1)),    (SAVE(218),NALIGN)
     *,           (SAVE(229),XNBE(1)),      (SAVE(232),YNBE(1))
```



```
     *,           (SAVE(235),ZNBE(1)),      (SAVE(258),X(1))
     *,           (SAVE(276),WE(1,1))
C     ZERO THE WORKING ARRAYS
      DO 5 I=1,700
    5 BLK(I)=0.
C     SET PAGE NO OF FIRST PAGE
   10       NPAGE = 1
        LOOP = LOOP + 1
        IF(LOOP.EQ.1) GO TO 20
       DO 11 I=1,10                                                A
   11  IF(NW(I).LT.0) NW(I) = 0
      WRITE(77) (DATA(I),I=1,350)
      CALL OUTDAT
      DO 15 I=1,350
   15 DATA(I)=0.                                                   A
        IF(LOOP.GT.IDENT) GO TO 20
        WRITE(6,80)
        GO TO 380
C     READ CONTROL INTEGERS INTO PROBLEM
   20       LOOP=1
            READ(5,30) (NTEGER(J),J=1,14)
C     MAKE END-OF-FILE CHECK
            IF(EOF(5)) 21, 22
   21       CALL EXIT
   22       CONTINUE
```

```
C        WRITE DATE, TIME AND SIZE OF UPCOMING STATISTICAL SET   B     INPUT
         CALL DATE(IDATE)                                              INPUT
         CALL TIME(ITIME)                                              INPUT
         WRITE (77) IDATE,ITIME,IDENT                            B     INPUT
   30             FORMAT(14I5)                                          INPUT
   40             IF(NMORE)60,60,50                                     INPUT
   50             NMO = 14 + NMORE                                      INPUT
                  READ(5,30)  (NTEGER(J),J=15,NMO)                      INPUT
C        WRITE HEADING AT TOP OF PAGE                                   INPUT
   60             CONTINUE                                              INPUT
                  NLINE=30                                              INPUT
C        READ AND WRITE TWO CARDS OF RUN INFORMATION                    INPUT
                  READ(5,80)                                            INPUT
   80             FORMAT(72H                                            INPUT
     1                           /72H                                   INPUT
     2                                              )                   INPUT
                  WRITE(6,80)                                           INPUT
            WRITE(6,1000)                                               INPUT
 1000       FORMAT(1H0,14X.29H** DATA INPUT FOR THIS RUN **//18X,       INPUT
     1      9HPARAMETER,6X,4HDATA/)                                     INPUT
C        READ CONTROL FLAGS FOR DATA STORAGE                     C      INPUT
         READ 475, (NST(I),I=1,15)                                      INPUT
         PRINT 480,(NST(I),I=1,15)                                      INPUT
 475     FORMAT(15I5)                                                   INPUT
 480     FORMAT(/1X,51HBLOCK DATA WILL BE STORED AT THE FOLLOWING NGUIDES- INPUT
        *,15I5)                                                  C      INPUT
C        CHECK FOR INDIVIDUAL FLOATING POINT DATA ENTRY                 INPUT
  100             IF(NP)250,250,110                                     INPUT
  110             DO 140 J = 1,NP                                       INPUT
                  READ(5,130) I,P(I)                                    INPUT
```



```
  130             FORMAT(I5,E15.7)                                      INPUT
            WRITE(6,1001) I,P(I)                                        INPUT
 1001       FORMAT(17X,I5,3X,E15.8)                                     INPUT
  140             CONTINUE                                              INPUT
                  IF(NTEGER(13).GT.0) WRITE(6,1003)                     INPUT
 1003       FORMAT(//20X,19H*******************/                        INPUT
     1      20X,19H*******************/20X,2H**,15X,2H**/               INPUT
```

```
     2    20X,19H**  BE IT KNOWN  **/
     3    20X,19H** THAT THIS RUN **/
     4    20X,19H**  WAS MADE IN  **/
     5    20X,19H**   AN OBLATE   **/
     6    20X,19H**  ENVIRONMENT  **/
     7    20X,2H**,15X,2H**/20X,19H*******************/
     8    20X,19H*******************)
C
C         CHECK IF NAVIGATION DATA IS IN INPUT FILE                    D
          IF(NPER.EQ.0) GO TO 155
C         READ AND SCALE NAVIGATION DATA
          DO 145 I=1,NPER
          READ(5,1004) NE(I),NM(I),DTL(I),DTN(I),DTM(I),USP(I),USV(I)
     1,UTP(I),UTV(I),SR(I),SRD(I),SO(I),SC1(I),SC2(I)
     2,NW(I)
          PRINT 460,NE(I),NM(I),DTL(I),DTN(I),DTM(I),USP(I),USV(I)
     1,UTP(I),UTV(I),SR(I),SRD(I),SO(I),SC1(I),SC2(I),NW(I)
  460     FORMAT(/1X,134H    NE    NM          DTL          DTN          DTM          USP
     1         USV         UTP         UTV          SR         SRD           SO
     2    SC1        SC2 NW       ,/1X,2I5,3F10.1,9F10.2,I3)
C         SCALE DATA
          DTL(I)=DTL(I)*60.
          DTN(I)=DTN(I)*60.
          DTM(I)=DTM(I)*60.
          USP(I)=USP(I)*1000.
          UTP(I)=UTP(I)*1000.
          SR(I)=SR(I)*1000.
          SO(I)=SO(I)/1000.
 1004     FORMAT(2I2,3F7.1,9F5.2,I2)
  145     CONTINUE                                                     D
          READ 1005,NALIGN, (TALIGN(I),I=1,NALIGN)                     E
 1005     FORMAT(I2,10E7.1)
          IF(NALIGN.LT.1) GO TO 150
          PRINT 465, NALIGN
  465     FORMAT(/1X,I2,44H ALIGNMENTS ARE SCHEDULED FOR THIS RUN AT-       )
          DO 146 I=1,NALIGN
  146     PRINT 470, TALIGN(I)
  470     FORMAT(/44X,F10.1)
  150     CONTINUE
  155     CONTINUE                                                     E
C
                  ICOM = 0
                  NFAM2=P(297)
C     CHECK IF COVARIANCE MATRIX IS TO BE READ IN
                  IF(NFAM2) 240,250,240
  240       CALL INPUTD
C      CHECK IF TABLE ENTRIES ARE TO BE MADE
  250             IF(NTABLE)380,380,260
```

```
  260          NT1(1) = NTCR
  270          DO 370 M = 1,NTABLE
  280          IF(NT(M))290,370,310
  290          NT1(M+1) = NT1(M)
  300          GO TO 370
  310          NT1(M+1) = NT1(M) + NT(M)
  320          NT2(M) = NT1(M) -1 + NT(M)
  330          NT11 = NT1(M)
  340          NT12 = NT2(M)
               READ(5,360) (P(J),J=NT11,NT12)
  360          FORMAT(7E10.7)
  370          CONTINUE
C      CALL INPUT WRITEOUT AND IC CALC ROUTINE
  380          CALL INAID
C      SET MANEUVER COMPUTATION FLAG
       P(10)=-(P(9) + 1.)
       NSTAT=0                                            F
       DATA(1)=LOOP
       DATA(321) = NFAM2
       PRINT 450, P(2)
  450  FORMAT(/1X,5HP(2) ,E15.6)
               NGUIDE=NTEGER(3)                           G
               LIGN=0
               ICOMP=1                                    G
C      SET TPI TIME INTO ERASABLE
               TIG(5)=TTPI
C    SET BRAKING GATE INDEX TO FIRST GATE
               NGATE=1
C      SET BRAKING FLAG TO ZERO
               NBRFL=0
C      ZERO THE Q AND SET IN IC
  390          DO 420 J = 1,N
  400          Q(J) = 0.0
  410          Y(J) = FIRSTY(J)
  420          CONTINUE
C ZERO THE WE MATRIX                                      H
       DO 421 I=1,18
       DO 421 J=1,27
       WE(I,J) = 0.0
  421  CONTINUE
C      SET UP VECTORS AND PERFORM INITIAL ALIGNMENT
       CALL SETY(Y)
       CALL ALIGN                                         H
C      CONSTRUCT ENVIRONMENT VECTOR
  425          CALL ICERR
  430          RETURN
               END
```

GNEXEC

ROUTINE GNEXEC

AREA A — GUIDANCE OVERLAY RETURN CHECK. IF NOVER FLAG IS OTHER THAN ZERO, GNEXEC HAS BEEN CALLED ON RETURN FROM THE GUIDANCE OVERLAY. VALUE OF NOVER (1,2,3,4) INDICATES PLACE IN GNEXEC WHICH CALLED GUIDANCE OVERLAY. EXECUTION OF THE GO TO STATEMENT RETURNS CONTROL TO STATEMENT FOLLOWING ONE THAT CALLED OVERLAY.

AREA B — LIGN FLAG IS INCREMENTED EACH TIME A PLATFORM ALIGNMENT IS PERFORMED. CHECK IS MADE TO DETERMINE IF LIGN IS EQUAL TO NUMBER OF ALIGNMENTS SCHEDULED ON INPUT. CHECK IS THEN MADE TO SEE IF THE NEXT ALIGN IS LESS THAN ONE INTEGRATION STEP AWAY.

AREA C — IF NO NAVIGATION PROCEDURE CARDS WERE READ IN ON INPUT, CONTROL IS TRANSFERRED OUT OF THE NAVIGATION CONTROL PORTION.

AREA D — BEGIN DO LOOP WHICH CYCLES THROUGH THE NAVIGATION CONTROL CARDS READ IN ON INPUT. IF CARD IS NOT ACTIVE, LOOP TURNS TO NEXT CARD.

AREA E — IF CARD IS ACTIVE FOR THIS NGUIDE ($NE(I) \neq 0$), CHECK IS MADE TO SEE IF A SENSOR IS ON. IF SENSOR IS OFF, CONTROL IS TRANSFERRED TO W-MATRIX INITIALIZATION SECTION TO SEE IF THIS CARD IS PRESENT ONLY TO RESET W.

AREA F — IF FIRST RENDEZVOUS MANEUVER IS NC1, TIME SINCE LAST MANEUVER IS DEFINED AS PROGRAM ELAPSED TIME. OTHERWISE, PROGRAM ELAPSED TIME MINUS PREVIOUS TIG.

AREA G — TIME TO NEXT MANEUVER IS TIG FOR CURRENT MANEUVER MINUS PROGRAM ELAPSED TIME.

AREA H — SEE IF TIME SINCE LAST MANEUVER IS GREATER THAN MINIMUM TIME TO BEGIN THIS PROCEDURE.

AREA I — SEE IF TIME TO NEXT MANEUVER IS STILL GREATER THAN MINIMUM TIME TO TERMINATE BEFORE MANEUVER.

AREA J — IF IT IS TIME TO TERMINATE THIS PROCEDURE, MANEUVER COMPUTATION FLAG IS SET = 1 TO INDICATE A MARKING PROCEDURE HAS TERMINATED AND THE GUIDANCE CONTROL SECTION CAN CALL THE GUIDANCE OVERLAY TO COMPUTE A BURN. TIME SINCE LAST MARK ON THIS PROCEDURE IS SET TO ZERO FOR NEXT CYCLE ON MONTE-CARLO SET.

AREA K — CHECK TO SEE IF THIS PROCEDURE CALLS FOR W TO BE RESET. OTHERWISE GO DIRECTLY TO MARKING SECTION AREAS M-P.

AREA L REINITIALIZE W:

SET SENSOR MARK COUNTERS TO ZERO.

SET THE CARD INITIALIZATION FLAG TO -1 INDICATING FOR FUTURE PASSES THAT REINITIALIZATION HAS BEEN DONE. THIS FLAG WILL BE RESET IN INPUT ON BEGINNING NEXT MONTE-CARLO CYCLE.

DEFINE TIME TAG ON W AS CURRENT ELAPSED PROGRAM TIME.

SET TIME OF LAST MARK ON THIS PROCEDURE EQUAL 0.

STORE THE CURRENT CARTESIAN ESTIMATED STATE FOR THE W-MATRIX ADVANCEMENT ROUTINE. ZERO OUT ELEMENTS OF W.

CALL FOR BLOCK PRINT AT THE REINITIALIZATION.

AREA M IF SENSORS WERE OFF AND THIS CARD IS ONLY TO RESET W, TURN TO NEXT CARD.

AREA N DEFINE THE TIME INTERVAL SINCE LAST MARK ON THIS PROCEDURE AS PROGRAM ELAPSED TIME MINUS TIME OF LAST MARK.

AREA O SINCE MARKING IS ABOUT TO TAKE PLACE, MANEUVER COMPUTATION FLAG IS RESET TO INDICATE THAT ESTIMATED STATE IS GOING TO BE UPDATED AND A NEW MANEUVER COMPUTATION WILL BE NECESSARY.

AREA P TAKING A MARK:

CHECK IF TIME SINCE LAST MARK IS GREATER THAN MINIMUM ALLOWED TIME BETWEEN MARKS ON THIS PROCEDURE. IF NOT, TURN TO NEXT CARD.

CALL P20 WITH CURRENT TIME, CURRENT ESTIMATED CARTESIAN STATE, AND SENSOR TYPE FLAG.

UPDATE THE MARK COUNTER. THE NM(I)=4 OPTION IS A LATE ADDITION COMBINING A RANGE AND OPTICS MARK. FOR THIS OPTION, BOTH THE RANGE AND OPTICS MARK COUNTERS ARE INCREMENTED.

SINCE THE CARTESIAN ESTIMATED STATE IS NOW UPDATED, IT IS NECESSARY TO RECONSTRUCT THE BETELGEUSE ESTIMATED STATE FOR INTEGRATION BY RK.

TRANSFER REVISED ESTIMATES OF SENSOR BIASES TO BETELGEUSE ESTIMATED VECTOR.

DEFINE TIME OF LAST MARK ON THIS PROCEDURE AS CURRENT PROGRAM TIME.

AREA Q IF A MARKING PROCEEDURE HAS JUST TERMINATED, THE ICOMP FLAG IS EQUAL 1. IN THIS CASE, AREA Q COMPUTES A MANEUVER BASED ON THE VALUE OF NGUIDE, ONCE FOR THE ACTUAL STATES AND ONCE FOR THE ESTIMATED. IF ICOMP≠1, CONTROL IS TRANSFERRED TO THE MANEUVER APPLICATION AREA (R) TO SEE IF IT IS TIME FOR A MANEUVER.

COMPUTE A MANEUVER (RECYCLE OR FINAL)

LOAD ACTUAL BETELGEUSE STATE INTO GUIDANCE OVERLAY LOCATIONS

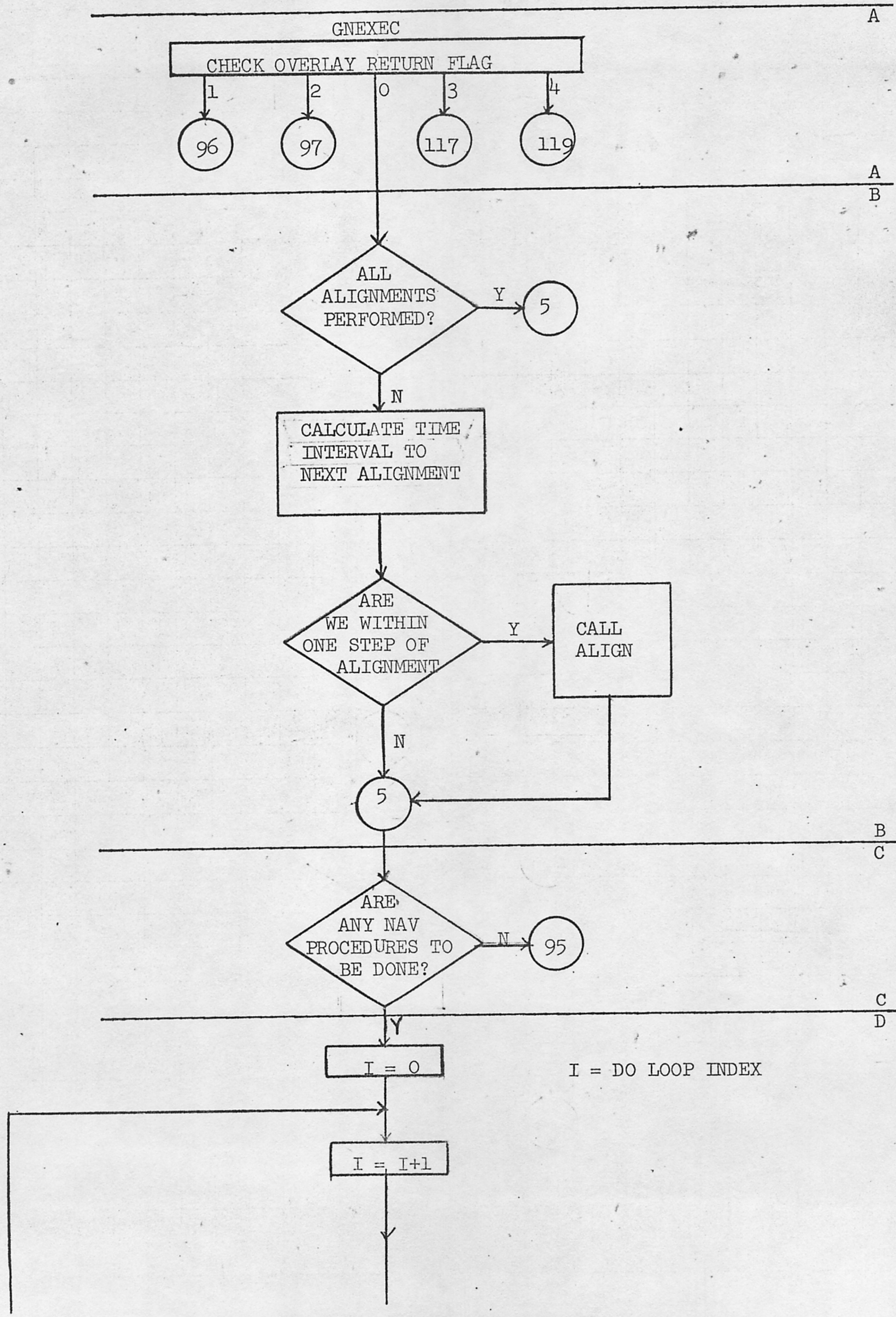
AREA Q — SET OVERLAY RETURN FLAG(NOVER) TO 1.
CALL GUIDANCE OVERLAY
CALL STORE1 TO STORE ACTUAL DELTAV. (CALLS TO THE STORE RESULT IN STORAGE OF INFORMATION ONLY IF NST(NGUIDE)=1 (SEE PAGE 5, CARD #4)).
CALL FOR BLOCK PRINT AT MANEUVER COMPUTATION.
SET OVERLAY RETURN FLAG, NOVER=2.
LOAD ESTIMATED STATES.
CALL GUIDANCE OVERLAY.
CALL STORE2 TO STORE ESTIMATED DELTAV.
RESET OVERLAY RETURN FLAG.
SET ICOMP=2 TO ADVERTISE THAT A MANEUVER HAS BEEN COMPUTED AND IS AVAILABLE.

AREA R — COMPUTE TIME TO GO UNTIL IGNITION.
SEE IF TGN IS LESS THAN 1 SECOND. IF IT IS, GO TO AREA R1 TO CALCULATE A MANEUVER IF THIS IS NOT ALREADY DONE.
IF TGN IS GREATER THAN 1 SECOND, SEE IF IT IS LESS THAN ONE INTEGRATION STEP. IF NOT, EXIT ROUTINE.
IF TGN IS LESS THAN 1 INTEGRATION STEP, SAVE THE DIFFERENCE BETWEEN TGN AND STEP FOR USE ON THE NEXT (SYNCH) PASS AFTER THE MANEUVER APPLICATION.
ALSO SAVE THE NOMINAL STEP SIZE.
DEFINE NEXT INTEGRATION STEP SIZE AS EQUAL TGN.
SET STEP SYNCH FLAG (ISTEP=1).
EXIT ROUTINE.

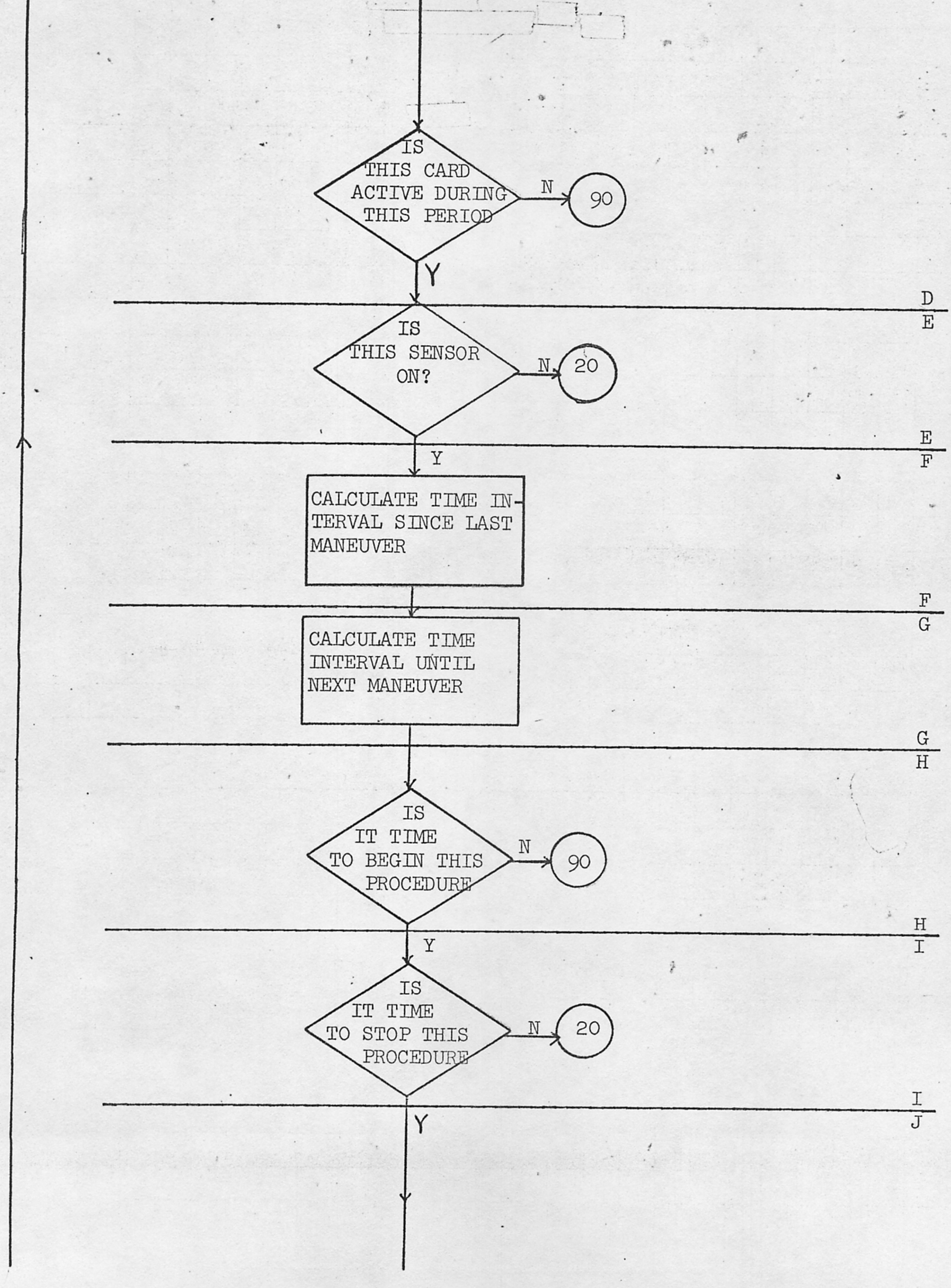
AREA RO — ON NEXT PASS AFTER ONE ON WHICH TGN WAS LESS THAN STEP, TGN WILL BE ZERO. AREA RO IS THEN VISITED AS A RESULT OF THE INSTRUCTION WHICH ASKS IF TGN IS LESS THAN 1 SECOND. STEP SYNCH FLAG IS INCREMENTED TO INDICATE NEXT PASS IS 'EVENING' STEP.
P(1) SET EQUAL TO T2

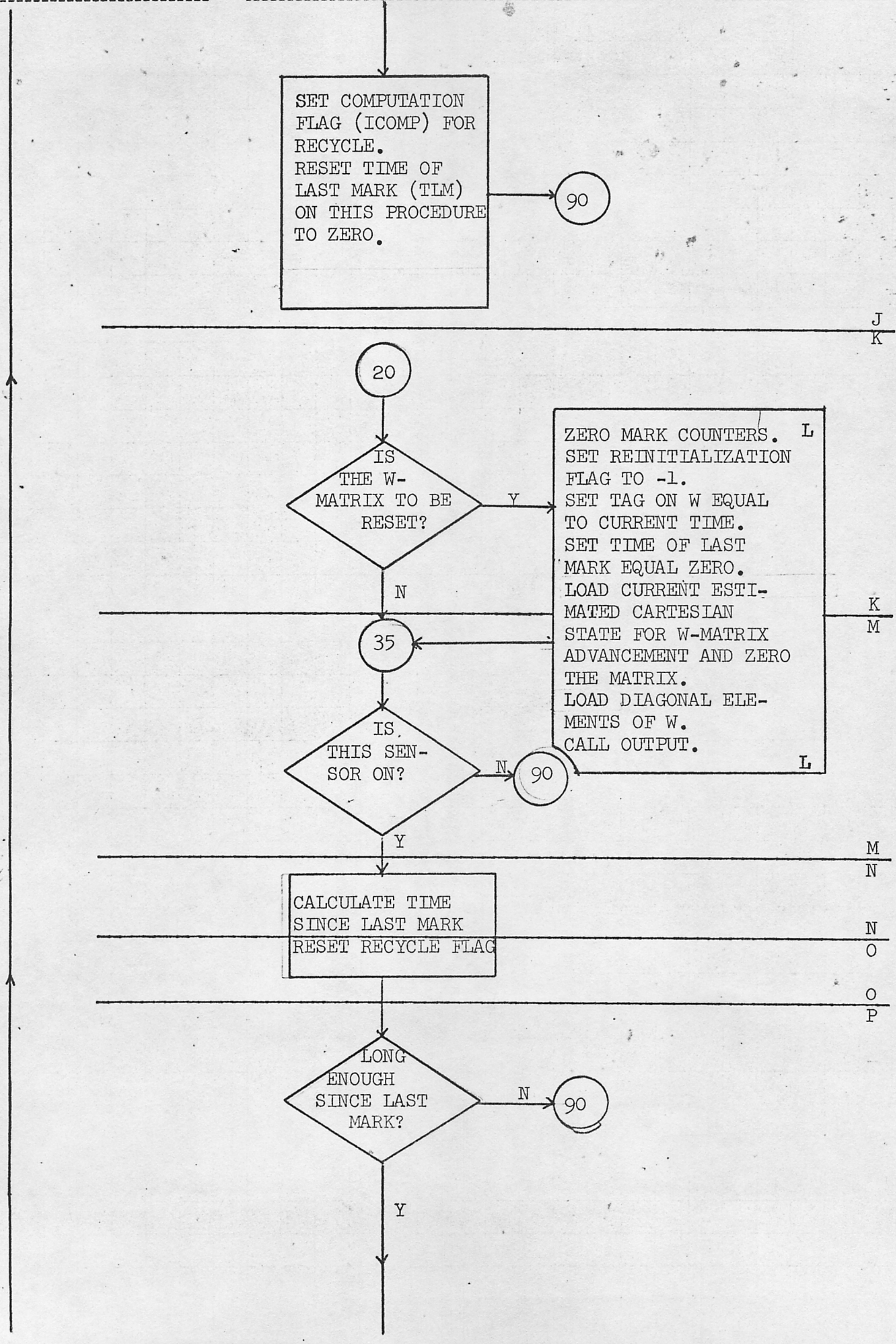
AREA R1 — CHECK IS MADE TO SEE IF MANEUVER HAS BEEN COMPUTED. IF SO, PROCEED DIRECTLY TO APPLICATION INSTRUCTIONS.
IF MANEUVER HAS NOT BEEN DEFINED, SEQUENCE OF COMPUTATIONAL INSTRUCTIONS IS PERFORMED IDENTICAL TO AREA Q.

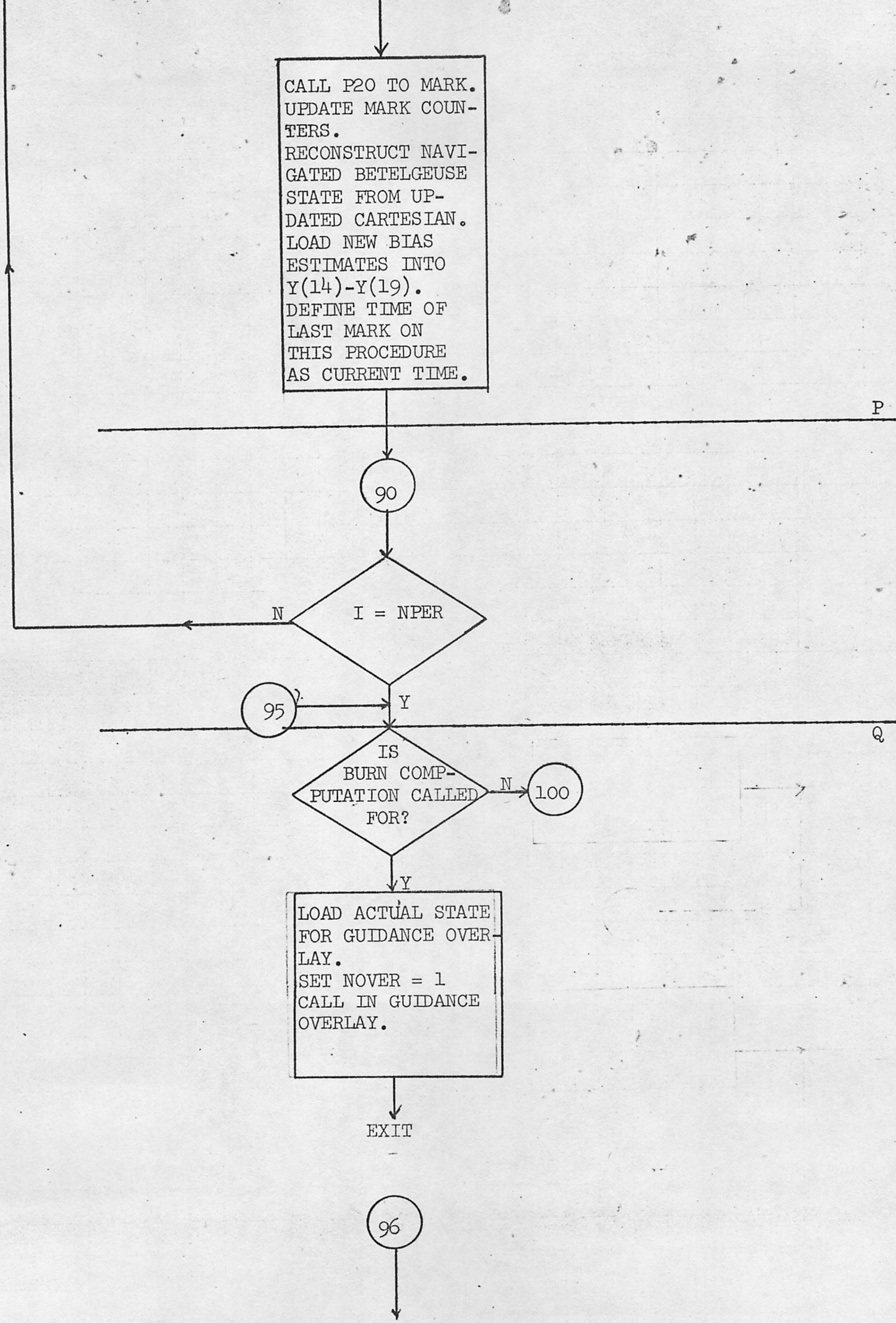
AREA R2 — INCREMENT NGUIDE
CALL DELTAV APPLICATION ROUTINE.
STORE STATES AT MANEUVER
EXIT ROUTINE.

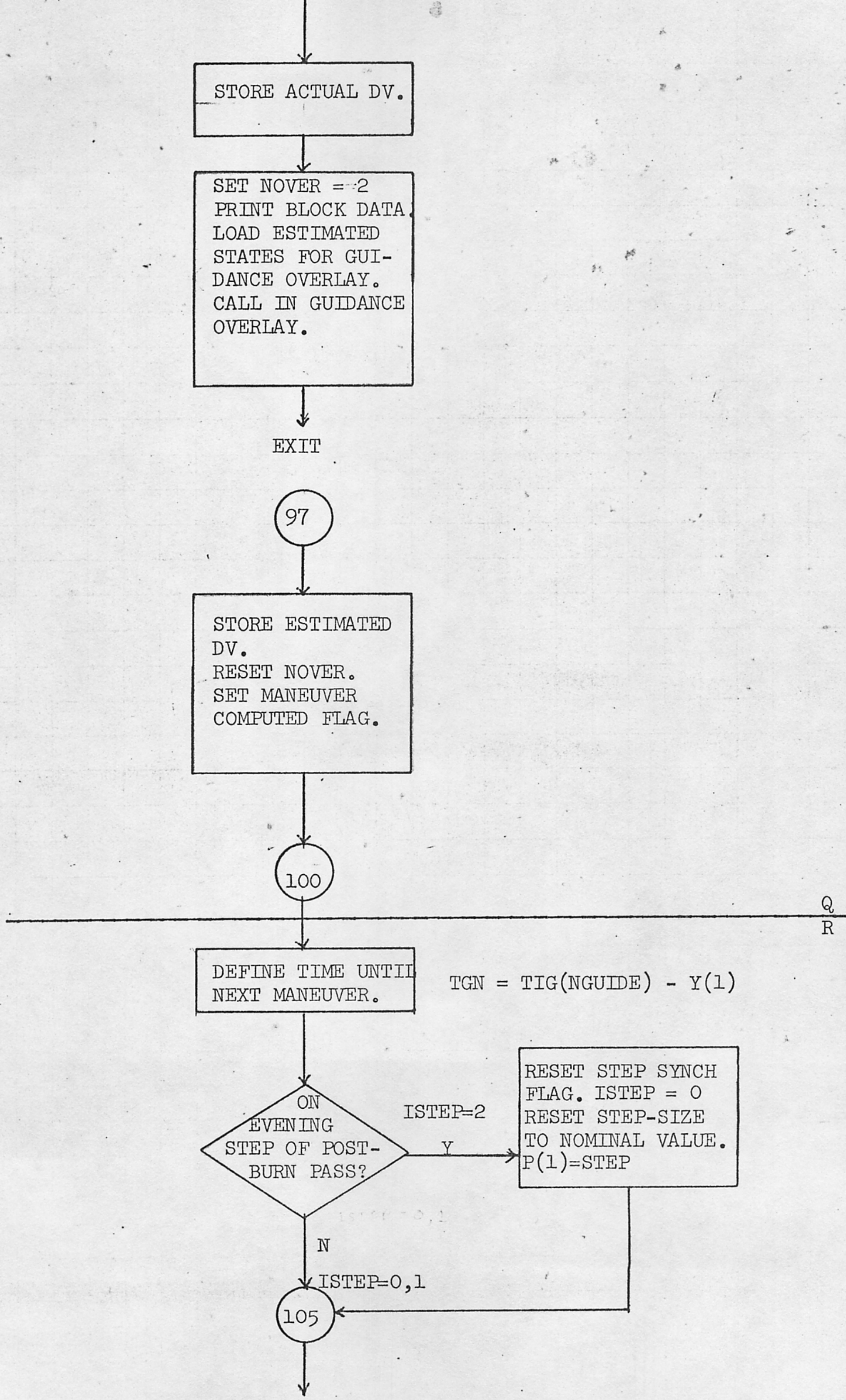

A
GNEXEC
CHECK OVERLAY RETURN FLAG
1
2
0
3
4
96
97
117
119
A
B
ALL ALIGNMENTS PERFORMED?
Y
5
N
CALCULATE TIME INTERVAL TO NEXT ALIGNMENT
ARE WE WITHIN ONE STEP OF ALIGNMENT
Y
CALL ALIGN
N
5
B
C
ARE ANY NAV PROCEDURES TO BE DONE?
N
95
C
D
Y
I = 0
I = DO LOOP INDEX
I = I+1

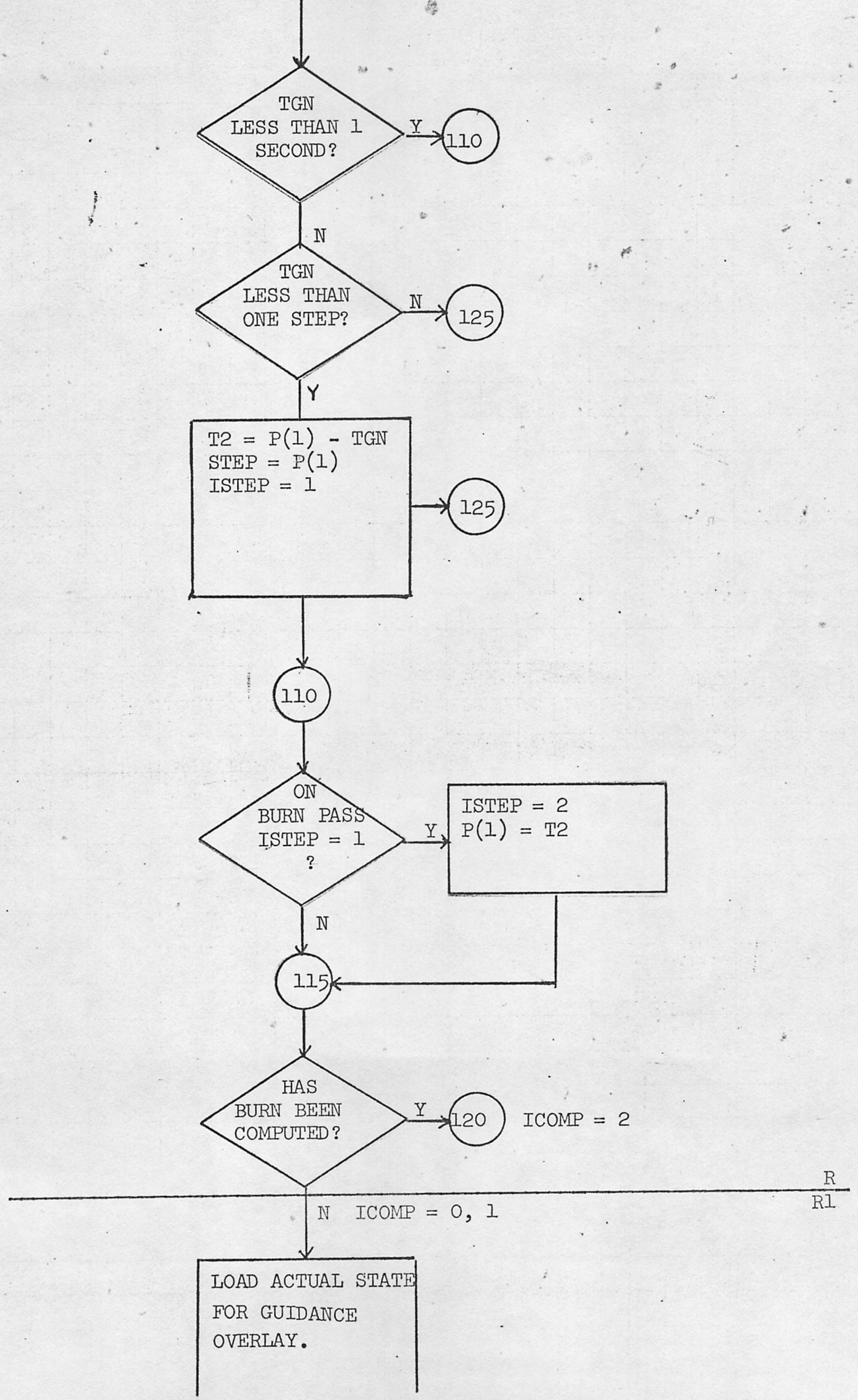
CHECKING Ith NAV PROCEDURE CARD




SET COMPUTATION FLAG (ICOMP) FOR RECYCLE.
RESET TIME OF LAST MARK (TLM) ON THIS PROCEDURE TO ZERO.
90
J
K
20
IS THE W-MATRIX TO BE RESET?
Y
N
L
ZERO MARK COUNTERS.
SET REINITIALIZATION FLAG TO -1.
SET TAG ON W EQUAL TO CURRENT TIME.
SET TIME OF LAST MARK EQUAL ZERO.
LOAD CURRENT ESTIMATED CARTESIAN STATE FOR W-MATRIX ADVANCEMENT AND ZERO THE MATRIX.
LOAD DIAGONAL ELEMENTS OF W.
CALL OUTPUT.
L
K
M
35
IS THIS SENSOR ON?
N
90
Y
M
N
CALCULATE TIME SINCE LAST MARK
RESET RECYCLE FLAG
N
O
O
P
LONG ENOUGH SINCE LAST MARK?
N
90
Y


CALL P20 TO MARK. UPDATE MARK COUNTERS. RECONSTRUCT NAVIGATED BETELGEUSE STATE FROM UPDATED CARTESIAN. LOAD NEW BIAS ESTIMATES INTO Y(14)-Y(19). DEFINE TIME OF LAST MARK ON THIS PROCEDURE AS CURRENT TIME.
P
90
I = NPER
N
Y
95
Q
IS BURN COMPPUTATION CALLED FOR?
N
100
Y
LOAD ACTUAL STATE FOR GUIDANCE OVERLAY. SET NOVER = 1 CALL IN GUIDANCE OVERLAY.
EXIT
96


STORE ACTUAL DV.
SET NOVER = 2
PRINT BLOCK DATA.
LOAD ESTIMATED
STATES FOR GUI-
DANCE OVERLAY.
CALL IN GUIDANCE
OVERLAY.
EXIT
97
STORE ESTIMATED
DV.
RESET NOVER.
SET MANEUVER
COMPUTED FLAG.
100
Q
R
DEFINE TIME UNTIL
NEXT MANEUVER.
TGN = TIG(NGUIDE) - Y(1)
ON
EVENING
STEP OF POST-
BURN PASS?
ISTEP=2
Y
RESET STEP SYNCH
FLAG. ISTEP = 0
RESET STEP-SIZE
TO NOMINAL VALUE.
P(1)=STEP
N
ISTEP=0,1
105



TGN LESS THAN 1 SECOND?
Y
110
N
TGN LESS THAN ONE STEP?
N
125
Y
T2 = P(1) - TGN
STEP = P(1)
ISTEP = 1
125
110
ON BURN PASS ISTEP = 1 ?
Y
ISTEP = 2
P(1) = T2
N
115
HAS BURN BEEN COMPUTED?
Y
120
ICOMP = 2
R
R1
N
ICOMP = 0, 1
LOAD ACTUAL STATE FOR GUIDANCE OVERLAY.

SET NOVER = 3
CALL IN GUIDANCE
OVERLAY

EXIT

117

STORE ACTUAL DV.

PRINTOUT BLOCK
DATA.
LOAD ESTIMATED
STATES FOR GUI-
DANCE OVERLAY.
SET NOVER = 4
CALL IN GUIDANCE
OVERLAY.

EXIT

119

STORE ESTIMATED
DELTA-V.
RESET NOVER.
SET ICOMP =2
INCREMENT NBRFL

R1

120

INCREMENT NGUIDE.
CALL MANEUVER AP-
PLICATION ROUTINE.
STORE STATES IF
REQUIRED.
RESET ICOMP FOR
NEXT MARKING
PERIOD.

R

125

RETURN

```
      SUBROUTINE GNEXEC                                                 GNEXEC
C                                                                       GNEXEC
C     SUBROUTINE TO CONTROL THE EXECUTION OF NAVIGATION PROCEDURES,     GNEXEC
C     GUIDANCE COMPUTATIONS AND MANEUVER APPLICATIONS                   GNEXEC
C                                                                       GNEXEC
      COMMON VAR                                                        GNEXEC
      DIMENSION VAR(5600), Y(100), DYDX(100), Q(100), FIRSTY(100)       GNEXEC
     *,         NTEGER(100), D(100), P(5000)                            GNEXEC
      EQUIVALENCE (VAR(1),Y(1))                                         GNEXEC
     *,           (VAR(101),DYDX(1))                                    GNEXEC
     *,           (VAR(201),Q(1))                                       GNEXEC
     *,           (VAR(301),FIRSTY(1))                                  GNEXEC
     *,           (VAR(401),NTEGER(1))                                  GNEXEC
     *,           (VAR(501),D(1))                                       GNEXEC
     *,           (VAR(601),P(1))                                       GNEXEC
      DIMENSION SAVE(950), BLK(700), DATA(350), COV(24,24)              GNEXEC
      EQUIVALENCE (P(350),SAVE(1))                                      GNEXEC
     *,           (P(1300),BLK(1))                                      GNEXEC
     *,           (P(4074),DATA(1))                                     GNEXEC
     *,           (P(4424),COV(1,1))                                    GNEXEC
      DIMENSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)     GNEXEC
     *,         ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)      GNEXEC
     *,         USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)     GNEXEC
     *,         SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)        GNEXEC
     *,         ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)    GNEXEC
     *,         X(18), WE(18,27)                                        GNEXEC
      EQUIVALENCE (SAVE(1),QQ(1)),          (SAVE(5),SIG(1))            GNEXEC
     *,           (SAVE(9),C(1)),           (SAVE(19),REFMAT(1,1))      GNEXEC
     *,           (SAVE(28),XNBN(1)),       (SAVE(31),YNBN(1))          GNEXEC
     *,           (SAVE(34),ZNBN(1)),       (SAVE(37),NE(1))            GNEXEC
     *,           (SAVE(47),NM(1)),         (SAVE(57),DTL(1))           GNEXEC
     *,           (SAVE(67),DTN(1)),        (SAVE(77),DTM(1))           GNEXEC
     *,           (SAVE(87),USP(1)),        (SAVE(97),USV(1))           GNEXEC
     *,           (SAVE(107),UTP(1)),       (SAVE(117),UTV(1))          GNEXEC
     *,           (SAVE(127),SR(1)),        (SAVE(137),SRD(1))          GNEXEC
     *,           (SAVE(147),SO(1)),        (SAVE(157),SC1(1))          GNEXEC
     *,           (SAVE(167),SC2(1)),       (SAVE(177),NW(1))           GNEXEC
     *,           (SAVE(187),TLM(1)),       (SAVE(197),NS(1))           GNEXEC
     *,           (SAVE(200),ZTZ(1)),       (SAVE(204),SZ(1))           GNEXEC
     *,           (SAVE(208),TALIGN(1)),    (SAVE(218),NALIGN)          GNEXEC
     *,           (SAVE(229),XNBE(1)),      (SAVE(232),YNBE(1))         GNEXEC
     *,           (SAVE(235),ZNBE(1)),      (SAVE(258),X(1))            GNEXEC
     *,           (SAVE(276),WE(1,1))                                   GNEXEC
```

```
      EQUIVALENCE (C(1),TW)                                                GNEXEC   45
     *,                (C(8),STEP)                                         GNEXEC   46
     *,                (C(9),T2)                                           GNEXEC
     *,                (C(10),TGN)                                         GNEXEC
      EQUIVALENCE (NTEGER(12),NPER)                                        GNEXEC   49
     *,                (NTEGER(29),NGUIDE)                                 GNEXEC   50
     *,                (NTEGER(30),ICOMP)                                  GNEXEC   51
     *,                (NTEGER(31),ISTEP)                                  GNEXEC   52
     *,                (NTEGER(32),NOVER)                                  GNEXEC   53
     *,                (NTEGER(33),LIGN)                                   GNEXEC   54
     *,                (NTEGER(35),NGATE)                                  GNEXEC   55
     *,                (NTEGER(36),NBRFL)                                  GNEXEC   56
```

SUBROUTINE   GNEXEC   CDC 6600 FTN V3.0-P308 OPT=1   08/29/72   11.32.27.

```
      DIMENSION DU(15), DV(15), DW(15), TFI(15)                            GNEXEC   57
      EQUIVALENCE (P(2141),TFI(1))                                         GNEXEC   58
     *,                (P(2156),DU(1))                                     GNEXEC   59
     *,                (P(2171),DV(1))                                     GNEXEC   60
     *,                (P(2186),DW(1))                                     GNEXEC   61
      IF(NOVER.GT.0) GO TO(96,97,117,119) NOVER                      A     GNEXEC   62
C                                                                          GNEXEC   63
C     CHECK IF TIME TO PERFORM AN ALIGNMENT                          B     GNEXEC   64
      IF(LIGN.GE.NALIGN) GO TO 5                                           GNEXEC   65
      KLIGN=LIGN+1                                                         GNEXEC   66
      DTLIGN=ABS(Y(1) - TALIGN(KLIGN))                                     GNEXEC   67
      IF(DTLIGN.GE.P(1)) GO TO 5                                           GNEXEC   68
      CALL ALIGN                                                           GNEXEC   69
    5 CONTINUE                                                       B     GNEXEC   70
C     CHECK IF ANY NAVIGATION PROCEDURES CALLED FOR ON THIS RUN            GNEXEC   71
      IF(NPER.EQ.0) GO TO 95                                         C     GNEXEC   72
C     CYCLE THROUGH LIST OF DEFINED PROCEDURES                       D     GNEXEC   73
      DO 90 I=1,NPER                                                       GNEXEC   74
C     CHECK IF PROCEDURE APPLICABLE TO THIS PREMANEUVER PERIOD              GNEXEC   75
      IF(NE(I).NE.NGUIDE) GO TO 90   GS Next card                    D     GNEXEC   76
C     CHECK IF THIS SENSOR IS OFF                                    E     GNEXEC   77
        IF(NM(I).EQ.0) GO TO20                                       E     AUTOW     1
C     DEFINE TIME SINCE LAST TIG                                     F     GNEXEC   79
      IF(NGUIDE.EQ.1) TGL=Y(1)                                             GNEXEC   80
      IF(NGUIDE.GT.1) TGL=Y(1) - TFI(NGUIDE-1)                       F     GNEXEC   81
```

```
C     DEFINE TIME UNTIL NEXT TIG
      TGN=TFI(NGUIDE) - Y(1)                                         G
C     CHECK IF IT IS TIME TO BEGIN THIS PROCEDURE                    H
      IF(TGL.LT.DTL(I)) GO TO 90                                     H
C     CHECK IF THIS PROCEDURE IS TO BE TERMINATED                    I
      IF(TGN.GT.DTN(I)) GO TO 20                                     I
C     SET MANEUVER COMPUTATION FLAG AND RESET W INITIALIZATION FLAG
      IF(ICOMP.NE.2) ICOMP=1                                         J
      TLM(I)=0.
      GO TO 90                                                       J
   20 CONTINUE                                                       K
C     CHECK IF W IS TO BE REINITIALIZED
      IF(NW(I).NE.0) GO TO 35                                        K
C     ZERO MARK COUNTERS                                             L
      NS(1)=0
      NS(2)=0
      NS(3)=0
C     SET W REINITIALIZED FLAG
      NW(I)=-1
C     DEFINE TIME TAG ON W
      TW=Y(1)
      TLM(I)=0.
C     STORE INITIAL UNIT VECTORS FOR ADVW
      CALL UVEC(Y(38),Y(39),Y(40),C(2))      NOT USED CAN
      CALL UVEC(Y(44),Y(45),Y(46),C(5))      BE DELETED
C     ZERO THE W-MATRIX
      DO 25 J=1,18
C     STORE STATE FOR ADVW
      X(J)=Y(37+J)
      DO 25 K=1,27
```

SUBROUTINE GNEXEC CDC 6600 FTN V3.0-P308 OPT=1 08/29/72 11.32.27.

```
   25 WE(J,K)=0.
C     LOAD THE DIAGONAL ELEMENTS OF W
      DO 30 J=1,3
      WE(J,J)=USP(I)
      WE(J+3,J+3)=USV(I)
      WE(J+6,J+6)=UTP(I)
      WE(J+9,J+9)=UTV(I)
```

```
   30 WE(J+14,J+14)=SO(I)                                                GNEXEC    119
      WE(13,13)=SR(I)                                                    GNEXEC    120
      WE(14,14)=SRD(I)                                                   GNEXEC    1[illegible]
      WE(17,17)=SC1(I)                                                   GNEXEC    1[illegible]
      WE(18,18)=SC2(I)                                                   GNEXEC    123
      CALL POPW                                                    L     GNEXEC    124
   35 CONTINUE                                                     M     GNEXEC    125
        IF(NM(I).EQ.0) GO TO 90                                    M     AUTOW       2
C     DEFINE TIME SINCE LAST MARK                                  N     GNEXEC    126
      DTLM=Y(1) - TLM(I) + 1.                                      N     GNEXEC    127
C     SINCE MARKING IN PROGRESS, RESET MANEUVER COMPUTATION FLAG         GNEXEC    128
      ICOMP=0                                                      O     GNEXEC    129
C     CHECK IF SUFFICIENT TIME HAS ELAPSED FOR ANOTHER MARK              GNEXEC    130
      IF(DTLM.LT.DTM(I)) GO TO 90                                  P     GNEXEC    131
C     CALL P20 TAKE AND INCORPORATE MARK                                 GNEXEC    132
      CALL P20(Y(1),Y(38),NM(I))                                         GNEXEC    133
C     UPDATE MARK COUNTER                                                GNEXEC    134
      INS=NM(I)                                                          GNEXEC    135
      IF(NM(I).EQ.4) NS(3)=NS(3) + 1                                     GNEXEC    136
      IF(NM(I).EQ.4) NS(2)=NS(2) + 1                                     GNEXEC    137
      IF(NM(I).LE.3) NS(INS)=NS(INS) + 1                                 GNEXEC    138
C     RECONSTRUCT NAVIGATED BETELGEUSE VECTOR AFTER HAVING TAKEN MARK    GNEXEC    139
      CALL CART2(Y(38),Y(2))                                             GNEXEC    140
      DO 85 J=1,6                                                        GNEXEC    141
      Y(J+13) = Y(J+49)                                                  GNEXEC    142
   85 CONTINUE                                                           GNEXEC    143
      TLM(I)=Y(1)                                                  P     GNEXEC    144
   90 CONTINUE                                                           GNEXEC    145
   95 CONTINUE                                                           GNEXEC    146
C     CHECK IF MANEUVER COMPUTATION FLAG IS SET                    Q     GNEXEC    147
      IF(ICOMP.NE.1) GO TO 100                                           GNEXEC    148
C     LOAD ENVIRONMENT, CALL COMPUTATIONS, SET RETURN FLAG               GNEXEC    149
      P(2001)=Y(1)                                                       GNEXEC    150
      DO 98 J=1,12                                                       GNEXEC    151
   98 P(2001+J)=Y(19+J)                                                  GNEXEC    152
      NOVER=1                                                            GNEXEC    153
      CALL DUMMY2                                                        GNEXEC    154
   96 CONTINUE                                                           GNEXEC    155
      CALL STORE1                                                        GNEXEC    156
      NOVER=2                                                            GNEXEC    157
      P(2001)=Y(1)                                                       GNEXEC    158
      CALL POPW                                                          GNEXEC    159
      DO 99 J=1,12                                                       GNEXEC    160
   99 P(2001+J)=Y(1+J)                                                   GNEXEC    161
      CALL DUMMY2                                                        GNEXEC    162
   97 CONTINUE                                                           GNEXEC    163
      CALL STORE2                                                        GNEXEC    164
      NOVER=0                                                            GNEXEC    165
```

```
C         INCREMENT NGUIDE
          IF(NBRFL.GT.0) NGATE=NGATE + 1
C         SET COMPUTATIONS PERFORMED FLAG
          ICOMP=2
  100     CONTINUE                                              Q
C         DEFINE TIME TO NEXT BURN                              R
          TGN=TFI(NGUIDE) - Y(1)
C         CHECK IF FINAL PASS THROUGH MANEUVER APPLICATION SEQUENCE
          IF(ISTEP.NE.2) GO TO 105
C         RESET ISTEP AND CHANGE TO ORIGINAL STEP-SIZE
          ISTEP=0
          P(1)=STEP
  105     CONTINUE
C         CHECK IF TIME FOR MANEUVER APPLICATION
          IF(ABS(TGN).LE.1.) GO TO 110
C         CHECK IF MANEUVER IS LESS THAN ONE STEP AWAY
          IF(ABS(TGN).GT.P(1)) GO TO 125
C         SET STEP-SIZE EQUAL TIME-TO-GO, STORE STEP
          T2=P(1) - TGN
          STEP=P(1)
          P(1)=TGN
          ISTEP=1
          GO TO 125
  110     CONTINUE                                              RO
          IF(ISTEP.NE.1) GO TO 115
          ISTEP=2
          P(1)=T2
  115     CONTINUE
C         CHECK IF MANEUVER HAS BEEN COMPUTED
          IF(ICOMP.EQ.2) GO TO 120                              RO
C         CALL  MANEUVER COMPUTATIONS FOR ENVIRONMENT AND NAVIGATED STATES
          P(2001)=Y(1)                                          R1
          DO 116 J=1,12
  116     P(2001+J)=Y(19+J)
          NOVER=3
C         LOAD GUIDANCE OVERLAY
                CALL DUMMY2
  117     CONTINUE
          CALL STORE1
          P(2001)=Y(1)
          CALL POPW
          DO 118 J=1,12
  118     P(2001+J)=Y(1+J)
```

```
      NOVER=4
C     LOAD GUIDANCE OVERLAY
            CALL DUMMY2
  119 CONTINUE
      CALL STORE2
      NOVER=0
      ICOMP=2
C     INCREMENT NGUIDE
      IF(NBRFL.GT.0) NGATE=NGATE + 1                          R1
  120 CONTINUE                                                R2
C     PERFORM MANEUVER
      NGUIDE=NGUIDE+1
```

UBROUTINE GNEXEC CDC 6600 FTN V3.0-P308 OPT=1 08/29/72 11.32.27.

```
      NV=NGUIDE - 1
      CALL DELTAV(DU(NV),DV(NV),DW(NV))
      CALL STORE3
      ICOMP=0                                                 R2
  125 CONTINUE
      RETURN
      END
```

DELTA-V

ROUTINE DELTAV

ROUTINE APPLIES MANEUVER BASED ON ESTIMATED STATE COMPUTATION TO ESTIMATED S/C STATE. ALSO COMPUTES EFFECT OF ACCELEROMETER SCALE FACTOR ERROR, PLATFORM MISALIGNMENT AND CUTOFF UNCERTAINTY ON APPLICATION TO ACTUAL STATE. VECTORS AND MATRICES INVOLVED IN THESE CALCULATIONS ARE:

$\underline{DXD}$	NOMINAL DELTAV IN LOCAL VERTICAL FRAME. THIS DELTAV IS COMPUTED FROM ESTIMATED STATE AND IS THE ONBOARD ESTIMATE OF THE APPLIED BURN.
$\underline{SD}$	VECTOR OF ACCELEROMETER SCALE FACTOR ERRORS.
$\underline{S}$	SCALE FACTOR DISTURBANCE MATRIX. should be correlated
$\underline{VN}$	VECTOR OF CUTOFF UNCERTAINTY ERROR.
$\underline{UX}, \underline{UY}, \underline{UZ}$	UNIT VECTORS OF THE ESTIMATED LOCAL VERTICAL FRAME.
$\underline{DUM}^1$	TRANSFORMATION FROM BRF TO ESTIMATED LOCAL VERTICAL FRAME.
$\underline{DUMT}^1$	TRANSFORMATION FROM ESTIMATED LOCAL VERTICAL FRAME TO BRF. (= $\underline{DUM}^T$)
$\underline{REFMAT}$	TRANSFORMATION FROM BRF TO ESTIMATED PLATFORM AXES.
$\underline{DVI}$	NOMINAL DELTAV IN BRF.
$\underline{DVSM}$	DELTAV IN ESTIMATED PLATFORM FRAME WITH SCALE FACTOR ERRORS APPLIED.
$\underline{DVSME}$	$\underline{DVSM} + \underline{VN}$
$\underline{GAMD}$	MATRIX OF PLATFORM DRIFT ERROR ANGLES.
$\underline{DUM}^2$	$\underline{S} \times \underline{REFMAT}$
$\underline{DUM}^3$	$\underline{GAMD} \times \underline{REFMAT}$
$\underline{DUMT}^2$	$\underline{REFMAT}^T \times \underline{GAMD}^T$
$\underline{DVIE}$	DELTAV IN BRF DISTURBED BY SCALE FACTOR ERROR, CUT-OFF UNCERTAINTY AND PLATFORM MISALIGNMENT.

$$\underline{DVIE} = \underline{REFMAT}^T \times \underline{GAMD}^T \times [\underline{S} \times \underline{REFMAT} \times \underline{DUMT} \times \underline{DXD} + \underline{VN}]$$

AREA A READ ARGUMENT DELTAV INTO OPERATING ARRAY.

AREA B IF NGUIDE IS STILL EQUAL TO ITS STARTING VALUE, THIS IS THE FIRST MANEUVER APPLICATION. IN THIS CASE, VISIT RANDOM NUMBER GENERATOR TO DEFINE SCALE FACTOR ERRORS FOR THIS CYCLE. OTHERWISE, PROCEED TO DEFINE CUTOFF ERROR IN AREA C.

AREA C DEFINE DIFFERENT RANDOM CUTOFF ERROR FOR EACH MANEUVER.

AREA D COMPUTE TRANSFORMATION FROM ESTIMATED LOCAL VERTICAL FRAME TO INERTIAL (BRF).
TRANSFORM LOCAL VERTICAL DELTAV'S TO BRF.

AREA E CONSTRUCT SCALE FACTOR MATRIX. THIS IS THE IDENTITY MATRIX WITH SCALE FACTOR ERRORS ADDED TO DIAGONAL.

AREA F <u>REFMAT</u> MATRIX IS TRANSFORM TO ESTIMATED PLATFORM AXES. EFFECT OF AREA F IS TO CONVERT BRF DELTAV TO ESTIMATED PLATFORM, MULTIPLY BY SCALE FACTOR ERRORS AND ADD CUT-OFF ERROR.

AREA G PLATFORM DRIFT ANGLES ARE D1,D2,D3. EFFECT OF AREA G IS TO TRANSFORM DELTAV TO ACTUAL PLATFORM FRAME AND RETURN FROM ACTUAL PLATFORM FRAME TO BRF.

AREA H CONVERT PERTURBED DELTAV TO INERTIAL FRAME.

AREA I ADVANCE W-MATRIX TO TIME OF BURN. THIS IS NECESSARY BE-CAUSE ESTIMATED STATE IS CHANGED BY APPLICATION OF DELTAV. SINCE <u>ADVW</u> USES PREVIOUS VALUE OF ESTIMATED STATE, THIS STATE <u>MUST</u> REFLECT THE MANEUVER. <u>X</u>, THE <u>ADVW</u> STORED STATE, WILL BE UPDATED IN AREA K.

AREA J CONSTRUCT A CARTESIAN FORM OF THE ACTUAL STATE.
ADD THE ACTUAL APPLIED DELTAV TO ACTUAL STATE.
ADD THE NOMINAL APPLIED DELTAV TO ESTIMATED STATE.

AREA K RECONSTRUCT ACTUAL BETELGEUSE STATE.
RECONSTRUCT ESTIMATED BETELGEUSE STATE.
UPDATE THE <u>ADVW</u> STORED STATE.

AREA L CONSTRUCT TRANSFORM TO LOCAL VERTICAL.
TRANSFORM ACTUAL APPLIED DELTAV TO LOCAL VERTICAL.
PRINT DELTAV APPLIED MESSAGE.

```
      SUBROUTINE DELTAV(DU,DV,DW)
C
      COMMON VAR
C
      DIMENSION VAR(5600), Y(100),DXD(3)
     *,P(5000), NTEGER(100), BLK(700)
     *,           VN(3),S(3,3)
     *,           UX(3), UY(3), UZ(3), DUM(3,3), DUMT(3,3)
     *,DVI(3), DVSM(3), DVSME(3), DVIE(3)
     *,GAMD(3,3), XE(12)
     *,           SAVE(950)
     *,REFMAT(3,3)
C     SUBROUTINE TO APPLY A LOCAL HORIZONTAL DELTA-V TO A BETEL VECTOR
C
C
      EQUIVALENCE (VAR(1),Y(1))
     *,             (VAR(401),NTEGER(1))
     *,             (VAR(601),P(1))
      EQUIVALENCE (P(350),SAVE(1))
     *,             (P(1300),BLK(1))
      DIMENSION X(18)
      EQUIVALENCE (SAVE(258),X(1))
      EQUIVALENCE (BLK(4),S(1,1)) ,(BLK(13),VN(1))
     *,(BLK(16),UX(1)), (BLK(19),UY(1)), (BLK(22),UZ(1))
     *,(BLK(25),DUM(1,1)), (BLK(34),DUMT(1,1)), (BLK(43),GAMD(1,1))
     *,(BLK(52),DVSM(1)), (BLK(55),DVSME(1)), (BLK(58),DVIE(1))
     *,(BLK(61),XE(1))
     *,(NTEGER(29),NGUIDE)
      EQUIVALENCE (SAVE(19),REFMAT(1,1))
     *,(Y(98),D1), (Y(99),D2), (Y(100),D3)
C
      COMMON/DELV/VARS,VARA,NFAMA,SD(3)
C
C     READ IN VELOCITIES                                   A
      DXD(1)=DU
      DXD(2)=DV
      DXD(3)=DW                                            A
C                                                          B
C     CHECK IF THIS IS THE FIRST MANEUVER OF THIS RUN
      NMAN=NGUIDE - NTEGER(3)
      IF(NMAN.NE.1) GO TO 5
C     CREATE SCALE FACTOR ERRORS FOR THIS RUN
```

```
      SD(2)=UNURN(0,NFAMA,1.,VARS)
      SD(3)=UNURN(0,NFAMA,1.,VARS)                                      B
    5 CONTINUE                                                          C
      DEFINE APPLICATION ERROR
      VN(1)=UNURN(0,NFAMA,0.,VARA)
      VN(2)=UNURN(0,NFAMA,0.,VARA)
      VN(3)=UNURN(0,NFAMA,0.,VARA)                                      C
C     COMPUTE DELTAV IN ASSUMED REFERENCE FRAME                         D
      CALL UVEC(Y(38),Y(39),Y(40),UX)
      CALL UCROSS(UX,Y(41),UZ)
      CALL UCROSS(UZ,UX,UY)
      CALL TRN(UX,UY,UZ,DUM,DUMT)
```



```
      CALL MATMUL(DUMT,DXD,DVI,3,3,1)                                   D
C     COMPUTE ACTUAL DELTAV APPLIED                                     E
C     COMPUTE SCALE FACTOR MATRIX
      DO 10 I=1,3
      DO 10 J=1,3
      S(I,J)=0.
      IF(I.EQ.J) S(I,J)=SD(I)
   10 CONTINUE                                                          E
      CALL MATMUL(S,REFMAT,DUM,3,3,3)                                   F
      CALL MATMUL(DUM,DVI,DVSM,3,3,1)
      CALL MATADD(DVSM,VN,DVSME,3,1,1)                                  F
C     COMPUTE PLATFORM MISALIGNMENT MATRIX                              G
      CALL MAT(D3,D2,D1,1,3,2,GAMD)
      CALL MATMUL(GAMD,REFMAT,DUM,3,3,3)
      CALL MATRAN(DUM,3,3,DUMT)                                         G
      CALL MATMUL(DUMT,DVSME,DVIE,3,3,1)                                H
      CALL ADVW(Y(1),Y(38))                                             I
C     CONSTRUCT CARTESIAN ENVIRONMENT VECTOR                            J
      CALL CART1(XE,Y(20))
      DO 15 J=1,3
      XE(J+3)=XE(J+3) + DVIE(J)
   15 Y(J+40)=Y(J+40) + DVI(J)                                          J
C     RECONSTRUCT BETELGEUSE STATES                                     K
      CALL CART2(XE,Y(20))
      CALL CART2(Y(38),Y(2))
```

```
      DO 12 I=1,18                                                      DELTV
      X(I) = Y(37+I)                                                    DELTV
   12 CONTINUE                                                    K     DELTV
      CALL TRN(UX,UY,UZ,DUM,DUMT)                                 L     DELTV
      CALL MATMUL(DUM,DVIE,DVI,3,3,1)                                   DELTV
      PRINT 100, (DXD(I),I=1,3), (DVI(I),I=1,3)                   L     DELTV
  100 FORMAT(/1X,35HTHE NAVIGATION BURN ESTIMATE IS DU=,F10.3, 5H  DV=, DELTV
     1F10.3, 5H  DW=,F10.3,/1X,35HTHE ACTUAL COMPONENTS WERE      DU=,  DELTV
     2F10.3, 5H  DV=,F10.3, 5H  DW=,F10.3)                              DELTV
      RETURN                                                            DELTV
      END                                                               DELTV
```

STORE 1

ROUTINE STORE1

SUBROUTINE STORES DATA AT TIME OF MANEUVER APPLICATION FOR MANEUVERS SELECTED BY THE DATA STORAGE CONTROL CARD, SEE PAGE 5, CARD #4.

AREA A — IF ROUTINE IS ENTERED AT STORE1, CALL IS FOR THE PURPOSE OF STORING DELTA-V'S FROM ACTUAL STATE COMPUTATION. SET N=26 TO SELECT 25th ELEMENT OF 31 ELEMENT MANEUVER DATA VECTOR. (N=26 SELECTS 25th ELEMENT BECAUSE FIRST 31 ELEMENT ARRAY STARTS AT DATA(2)).

AREA B — IF ROUTINE IS ENTERED AT STORE2, CALL IS FOR PURPOSE OF STORING DELTA-V'S FROM ESTIMATED STATE COMPUTATION. SET N=29 TO SELECT 28th ELEMENT OF 31 ELEMENT ARRAY.

AREA C — CHECK NST(NGUIDE)=1 TO SEE IF DATA FROM THIS MANEUVER SHOULD BE STORED. IF NOT, EXIT ROUTINE.

AREA D — NSTAT INDEX KEEPS TRACK OF THE NUMBER OF 31-ELEMENT DATA VECTORS SO FAR STORED. THIS NUMBER MAY NOT EXCEED 8.
K IS THE STARTING LOCATION IN THE DATA ARRAY FOR THE NEW INFORMATION.
CALL TO VEC LOADS DU, DV, DW INTO DATA(K,K+1,K+2).

AREA E — IF ROUTINE IS ENTERED AT STORE3, CALL IS FOR THE PURPOSE OF STORING ACTUAL AND ESTIMATED STATES AT TIME OF BURN APPLICATION. SINCE NGUIDE HAS ALREADY BEEN INCREMENTED IN GNEXEC, DEFINE K AS NGUIDE-1, FOR THE MANEUVER JUST PERFORMED.
CHECK IF DATA FROM THIS MANEUVER SHOULD BE STORED. IF NOT, EXIT ROUTINE.

AREA F — CREATE CARTESIAN FORM OF ACTUAL BETELGEUSE STATE.
DEFINE STARTING LOCATION IN DATA ARRAY.
LOAD ACTUAL AND ESTIMATED CARTESIAN STATES.

AREA G — INCREMENT NSTAT TO SHOW ANOTHER STORED MANEUVER.

AREA H — STORE TIME OF MANEUVER IN LAST ELEMENT OF 31 ELEMENT ARRAY JUST WRITTEN.

```
      SUBROUTINE STORE1
C
C     SUBROUTINE TO STORE ASSORTED PARAMETERS IN THE DATA ARRAY
C
      COMMON VAR
C
      DIMENSION VAR(5600)
     *,            Y(100)
     *,            P(500C)
     *,            NTEGER(100)
     *,            DATA(350)
     *,            NST(15)
     *,            DU(15),DV(15),DW(15)
     *,            XE(12)
C
      EQUIVALENCE (VAR(1),Y(1))
     *,           (VAR(401),NTEGER(1))
     *,           (VAR(601),P(1))
     *,           (VAR(934),NSTAT)
     *,           (VAR(935),NST(1))
     *,           (P(4074),DATA(1))
     *,           (NTEGER(29),NGUIDE)
     *,           (P(2156),DU(1)),(P(2171),DV(1)),(P(2186),DW(1))
C
      N=26                                                          A
      GO TO 5                                                       A
      ENTRY STORE2                                                  B
      N=29                                                          B
    5 CONTINUE
      IF(NST(NGUIDE).NE.1) GO TO 30                                 C
      K=NSTAT*31 + N                                                D
      CALL VEC(DU(NGUIDE),DV(NGUIDE),DW(NGUIDE),DATA(K))
      GO TO 30                                                      D
      ENTRY STORE3                                                  E
      K=NGUIDE - 1
      IF(NST(K).NE.1) GO TO 30                                      E
      CALL CART1(XE,Y(20))                                          F
      J=NSTAT*31 + 1
      DO 20 I=1,12
      DATA(I+J)=XE(I)
   20 DATA(I+J+12)=Y(I+37)                                          F
      NSTAT=NSTAT + 1                                               G
      J=NSTAT*31 + 1                                                H
      DATA(J)=Y(1)                                                  H
   30 CONTINUE
      RETURN
      END
```

OUTDAT

ROUTINE OUTDAT

THIS ROUTINE DUMPS THE DATA(350) ARRAY AT THE END OF EACH MONTE CARLO CYCLE. THIS PROVIDES A VISUAL INSPECTION OF THE DATA BEING WRITTEN ONTO THE LOCAL MASS STORAGE FILE.

AREA A WRITE OUT THE CURRENT VALUE OF THE MONTE CARLO RUN INDEX.

AREA B DUMP THE 8 MANEUVER DATA VECTORS.

AREA C SAME AS AREA A.

AREA D DUMP THE 10 PLATFORM ALIGNMENT DATA VECTORS.

AREA E DUMP THE USER SPECIFIED PORTION OF DATA.

```
      SUBROUTINE OUTDAT                                                 DELTV     140
C     SUBROUTINE TO OUTPUT STORED DATA FROM EACH STATISTICS RUN         DELTV     141
C                                                                       DELTV     142
      COMMON VAR                                                        DELTV     143
      DIMENSION VAR(5600), P(5000), DATA(350)                           DELTV     144
      EQUIVALENCE (VAR(601),P(1))                                       DELTV     145
      EQUIVALENCE (P(4074),DATA(1))                                     DELTV     146
C                                                                       DELTV     147
      NRUN=DATA(1)                                               A      DELTV     148
      WRITE(6,100) NRUN                                          A      DELTV     149
C                                                                       DELTV     150
      DO 5 I=1,31                                                    B  DELTV     151
    5 WRITE(6,105) DATA(I+  1),DATA(I+ 32),DATA(I+ 63),DATA(I+ 94)      DELTV     152
   5*,            DATA(I+125),DATA(I+156),DATA(I+187),DATA(I+218) B     DELTV     153
C                                                                       DELTV     154
      WRITE(6,110) NRUN                                          C      DELTV     155
C                                                                       DELTV     156
      DO 10 I=1,7                                                D      DELTV     157
   10 WRITE(6,115) DATA(I+250),DATA(I+257),DATA(I+264)                  DELTV     158
     *,            DATA(I+271),DATA(I+278),DATA(I+285)                  DELTV     159
     *,            DATA(I+292),DATA(I+299),DATA(I+306)                  DELTV     160
     *,            DATA(I+313)                                   D      DELTV     161
C                                                                       DELTV     162
      WRITE(6,120) (DATA(I),I=321,350)                           E      DELTV     163
  100 FORMAT(1H1,50X,32HVECTOR/MANEUVER DATA FOR RUN NO.,I4,10H FOLLOWS DELTV     164
     *-  ,/)                                                            DELTV     165
  105 FORMAT(1X,8E15.6)                                                 DELTV     166
  110 FORMAT(/50X,29HALIGNMENT HISTORY FOR RUN NO.,I4,10H FOLLOWS-  ,/) DELTV     167
  115 FORMAT(1X,10E13.4)                                                DELTV     168
  120 FORMAT(/44X,47HASSORTED OTHER JUNK KNOWN ONLY TO USER FOLLOWS-,   DELTV     169
     *3(/1X,8E15.6),/1X,6E15.6)                                         DELTV     170
      RETURN                                                            DELTV     171
      END                                                               DELTV     172
```

PØPØUT

*

ROUTINE POPOUT

SUBROUTINE HAS TWO FUNCTIONS:
(1) COMPUTES ACTUAL, ESTIMATED AND MEASURED RELATIVE PARAMETERS. MEASURED RELATIVE PARAMETERS ARE SUPPLIED TO NAVIGATION FILTER DURING MARKING PROCESS. POPOUT MUST BE VISITED PERIODICALL WHENEVER MARKING IS IN PROGRESS, EVEN IF NO PRINTED OUTPUT IS DESIRED.
(2) COMPUTES, ORGANIZES AND PRINTS OUT THE AAP STANDARD DATA BLOCK.

AREA A — IF ENTRY IS THROUGH CALL TO POPOUT, IT IS FOR THE NORMAL PURPOSES DESCRIBED ABOVE. DEPENDING ON VALUE OF P(9) AND ELAPSED TIME SINCE LAST EXECUTION OF THE PRINT INSTRUCTIONS, PRINT MAY OR MAY NOT RESULT. IF ENTRY IS THROUGH CALL TO POPW, IT IS FOR THE PURPOSE OF A BLOCK DATA PRINT AT A TIME OF MANEUVER COMPUTATION OR RESETTING W. IN THIS CASE, PRINT IS DESIRED REGARDLESS OF THE TIME SINCE LAST PRINT. SETTING OF NSKIP=1 CAUSES ELAPSED TIME CHECK AT PRINT INSTRUCTIONS TO BE DISABLED.

AREA B — CALLS TO SETUP CREATE AN OUTPUT FORM OF THE BETELGEUSE VECTOR FOR ESTIMATED AND ACTUAL STATES.

AREA C — ERROR IN SENSOR BIAS ESTIMATE IS DEFINED AS ESTIMATED MINUS ACTUAL. ANGLE BIASES ERRORS ARE CONVERTED TO MILLIRADIANS BEFORE OUTPUT.

AREA D — CREATE A CARTESIAN FORM OF THE ACTUAL BETELGEUSE STATE.

AREA E — MEASUREMENT FRAME IS 'NATURAL GEOMETRY' FRAME OF RELATIVE MEASUREMENTS.

AREA F — MFTMAT IS THE TRANSFORMATION MATRIX FROM BRF CARTESIAN FRAME TO LINE-OF-SIGHT (MEASUREMENT) FRAME. FOR PURPOSES OF COORDINATE TRANSFORMATION, THE W-MATRIX TRANSFORMS AS AN ERROR VECTOR (REFERENCE 1). BY THE DEFINITION OF THE ESTIMATED CARTESIAN STATE:

$$\underline{X}_N = [\underline{R}_{S/C}, \underline{V}_{S/C}, \underline{R}_{TGT}, \underline{V}_{TGT}, \underline{K}_S]$$

$\underline{X}_N$ = ESTIMATED STATE $\quad$ $\underline{K}_S$= ESTIMATED SENSOR BIASES

THE ERROR VECTOR IS AN ARRAY OF STATE ERRORS IN THE SAME SEQUENCE. SINCE THE POSITIONS AND VELOCITIES ARE ALL EXPRESSED IN THE SAME BRF FRAME, THEY HAVE THE SAME TRANSFORMATION TO THE MEASUREMENT FRAME:

$$\underline{MFTRN} = \begin{bmatrix} \underline{MFTMAT} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & \underline{MFTMAT} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & \underline{MFTMAT} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & \underline{MFTMAT} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & I_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & I_{3x3} \end{bmatrix}$$

WHERE MFTMAT IS THE MATRIX WHOSE ROWS ARE THE UNIT VECTORS OF THE MEASUREMENT FRAME IN THE BRF. SINCE MFTRN TRANSFORMS AN ERROR VECTOR, IT TRANSFORMS THE W-MATRIX. ALL SUBSEQUENT COMPUTATIONS INVOLVING THE W-MATRIX WILL USE THE MEASUREMENT FRAME W, DENOTED WM.

AREA G TRANSFORM W TO MEASUREMENT FRAME: $\underline{WM} = \underline{MFTRN} \times \underline{W}$

AREA H BY THE DEFINITION OF W, $WW^T=E$, THE COVARIANCE OF STATE ERRORS. AS THE DIAGONAL ELEMENTS OF WM ARE THE VARIANCES OF STATE ERRORS, THEIR SQUARE-ROOT IS THE STANDARD DEVIATION. AREA H COMPUTES THE DIAGONAL ELEMENTS OF $\underline{WM}\,WM^T$ AND FINDS THEIR SQUARE ROOTS.

AREA I COMPUTE THE COVARIANCE OF RELATIVE STATE ERRORS. LET

$$\underline{r} = \underline{R}_{TGT} - \underline{R}_{S/C}$$

$$\underline{v} = \underline{V}_{TGT} - \underline{V}_{S/C}$$

THEN

$$\begin{bmatrix} \underline{r} \\ \underline{v} \end{bmatrix} = \begin{bmatrix} -I_{3x3} & 0_{3x3} & I_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & -I_{3x3} & 0_{3x3} & I_{3x3} & 0_{3x3} & 0_{3x3} \end{bmatrix} \underline{X}_N$$

$$= [\underline{OK}]\ \underline{X}_N$$

FROM WHICH IT NECESSARILY FOLLOWS THAT

$$\underline{RWM}_{6x18} = \underline{OK}_{6x18} \times \underline{WM}_{18x18}$$

IS THE W-MATRIX OF RELATIVE STATE ERRORS, AND

$$\underline{REM} = \underline{RWM} \times \underline{RWM}^T$$

IS THE COVARIANCE OF RELATIVE ERRORS.

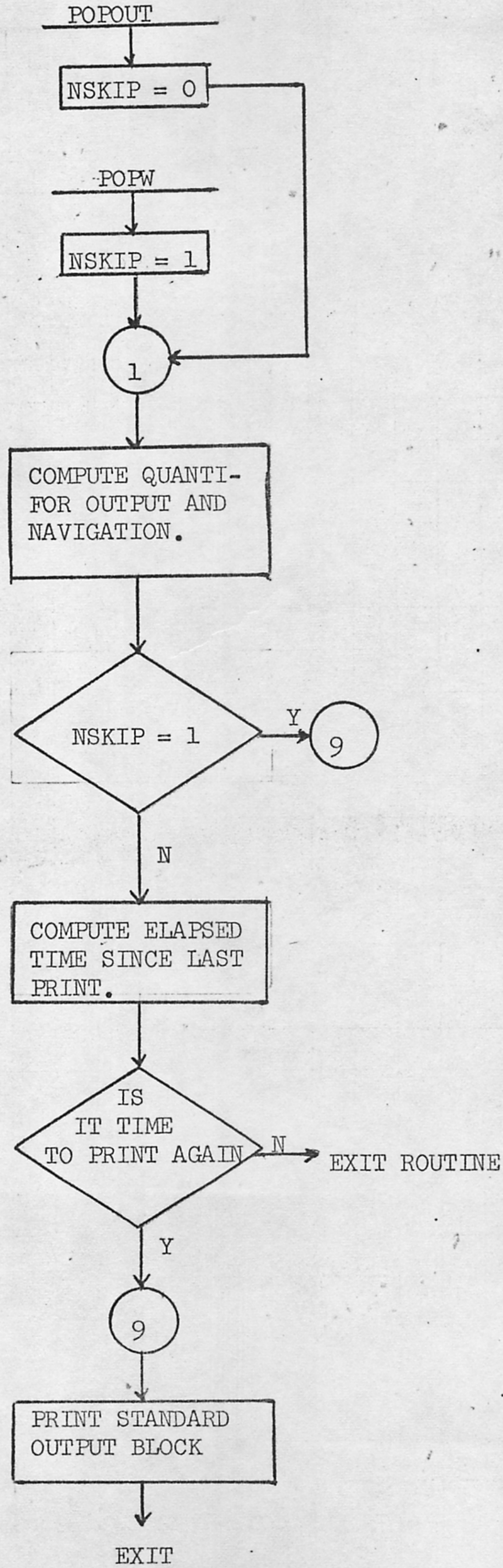
AREA K — CURRENT CARTESIAN ERROR VECTOR IS ROTATED TO MEASUREMENT FRAME.
CURRENT RELATIVE ERROR VECTOR IS COMPUTED.

AREA L — COMPUTES RELATIVE QUANTITIES FOR NAVIGATION AND OUTPUT.
CALL REF WITH ESTIMATED CARTESIAN S/C STATE TO DEFINE ACTUAL LOCAL VERTICAL UNIT VECTORS, ACTUAL NAV BASE UNIT VECTORS, ESTIMATED LOCAL VERTICAL VECTORS AND ESTIMATED NAV BASE VECTORS. CURRENT NAV BASE UNIT VECTORS ARE DEFINED AS THE LINE-OF-SIGHT FRAME UNIT VECTORS (SAME AS MEASUREMENT FRAME).
CALL REL WITH ACTUAL STATE AND ACTUAL NAV BASE VECTORS TO COMPUTE R, RDOT, AZ, EL , ACTUAL RELATIVE PARAMETERS. AZ, EL ARE ANGLES DEFINED WITH RESPECT TO NAV BASE UNIT VECTORS, NOT LOCAL VERTICAL.
CALL GARBAGE TO CREATE MEASURED RELATIVE PARAMETERS FROM ACTUAL RELATIVE PARAMETERS. RQE VECTOR USED FOR THIS COMPUTATION IS OVERWRITTEN WITH NEXT INSTRUCTION. THIS CALL TO GARBAGE ADDS NOISE TO RQE VECTOR AND STORES IT IN QQ.
CALL REL AGAIN WITH ACTUAL STATE AND ACTUAL LOCAL VERTICAL UNIT VECTORS TO DEFINE ACTUAL VALUE OF LOCAL VERTICAL AZ AND EL.
CALL REL AGAIN WITH ESTIMATED STATE AND ESTIMATED LOCAL VERTICAL UNIT VECTORS TO DEFINE ESTIMATED VALUE OF LOCAL VERTICAL AZ AND EL, ESTIMATED R, RDOT.
LOAD MEASURED VALUES INTO OUTPUT ARRAY.
CONVERT OUTPUT ANGLES TO DEGREES.

AREA M — CONVERT BIAS SIGMA FROM AREA H COMPUTATION TO MILLIRADIANS.

AREA N — FORGET IT. NOONE SEEMS SURE WHAT THIS MEANS OR IF IT IS CORRECT. IT DOES NOT SEEM TO BEAR ON THE OPERATION OF THE FILTER AND I HAVE BEEN UNABLE TO EXTRACT ANYTHING USEFUL FROM IT.

AREA O — IF ENTRY WAS THROUGH POPW, GO AROUND PRINT INTERVAL CHECK.


POPOUT
NSKIP = 0
POPW
NSKIP = 1
1
COMPUTE QUANTI- FOR OUTPUT AND NAVIGATION.
NSKIP = 1
Y
9
N
COMPUTE ELAPSED TIME SINCE LAST PRINT.
IS IT TIME TO PRINT AGAIN
N
EXIT ROUTINE
Y
9
PRINT STANDARD OUTPUT BLOCK
EXIT

```
      SUBROUTINE POPOUT                                               POPOUT   2
C                                                                     POPOUT   3
C     OUTPUT SUBROUTINE FOR NAVIGATED STATE                           POPOUT   4
C                                                                     POPOUT   5
C     FOLLOWING AREA IS BLANK COMMON FOR BETELGEUSE                   POPOUT   6
      COMMON VAR                                                      POPOUT   7
C     +++++                                                           POPOUT   8
      REAL MFTMAT, MFTRN                                              POPOUT   9
C     +++++                                                           POPOUT  10
      DIMENSION VAR(5600), Y(100), DYDX(100), Q(100), FIRSTY(100)     POPOUT  11
     *,        NTEGER(100), D(100), P(5000)                           POPOUT  12
      EQUIVALENCE (VAR(1),Y(1))                                       POPOUT  13
     *,          (VAR(101),DYDX(1))                                   POPOUT  14
     *,          (VAR(201),Q(1))                                      POPOUT  15
     *,          (VAR(301),FIRSTY(1))                                 POPOUT  16
     *,          (VAR(401),NTEGER(1))                                 POPOUT  17
     *,          (VAR(501),D(1))                                      POPOUT  18
     *,          (VAR(601),P(1))                                      POPOUT  19
      DIMENSION SAVE(950), BLK(700), DATA(350), COV(24,24)            POPOUT  20
      EQUIVALENCE (P(350),SAVE(1))                                    POPOUT  21
     *,          (P(1300),BLK(1))                                     POPOUT  22
     *,          (P(4074),DATA(1))                                    POPOUT  23
     *,          (P(4424),COV(1,1))                                   POPOUT  24
      DIMENSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)   POPOUT  25
     *,        ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)     POPOUT  26
     *,        USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)    POPOUT  27
     *,        SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)       POPOUT  28
     *,        ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)   POPOUT  29
     *,        X(18), WE(18,27)                                       POPOUT  30
     *,        XLVE(3),YLVE(3),ZLVE(3),XLVN(3),YLVN(3),ZLVN(3)        NOSHIT   1
      EQUIVALENCE (SAVE(1),QQ(1)),          (SAVE(5),SIG(1))          POPOUT  31
     *,          (SAVE(9),C(1)),            (SAVE(19),REFMAT(1,1))    POPOUT  32
     *,          (SAVE(28),XNBN(1)),        (SAVE(31),YNBN(1))        POPOUT  33
     *,          (SAVE(34),ZNBN(1)),        (SAVE(37),NE(1))          POPOUT  34
     *,          (SAVE(47),NM(1)),          (SAVE(57),DTL(1))         POPOUT  35
     *,          (SAVE(67),DTN(1)),         (SAVE(77),DTM(1))         POPOUT  36
     *,          (SAVE(87),USP(1)),         (SAVE(97),USV(1))         POPOUT  37
     *,          (SAVE(107),UTP(1)),        (SAVE(117),UTV(1))        POPOUT  38
     *,          (SAVE(127),SR(1)),         (SAVE(137),SRD(1))        POPOUT  39
     *,          (SAVE(147),SO(1)),         (SAVE(157),SC1(1))        POPOUT  40
     *,          (SAVE(167),SC2(1)),        (SAVE(177),NW(1))         POPOUT  41
     *,          (SAVE(187),TLM(1)),        (SAVE(197),NS(1))         POPOUT  42
```

```
     *,            (SAVE(208),TALIGN(1)),       (SAVE(218),NALIGN)               POPOUT    44
     *,            (SAVE(229),XNBE(1)),         (SAVE(232),YNBE(1))              POPOUT    45
     *,            (SAVE(235),ZNBE(1)),         (SAVE(258),X(1))                 POPOUT    46
     *,            (SAVE(276),WE(1,1))                                           POPOUT    47
     *,            (NTEGER(30),ICOMP)                                            POPOUT    48
      EQUIVALENCE (SAVE(762),XLVE(1))                                            NOSHIT     2
     *,          (SAVE(765),YLVE(1))                                             NOSHIT     3
     *,          (SAVE(768),ZLVE(1))                                             NOSHIT     4
     *,          (SAVE(771),XLVN(1))                                             NOSHIT     5
     *,          (SAVE(774),YLVN(1))                                             NOSHIT     6
     *,          (SAVE(777),ZLVN(1))                                             NOSHIT     7
C                                                                                POPOUT    49
```

```
SUBROUTINE   POPOUT                     CDC 6600 FTN V3.0-P308 OPT=1  08/29/72  11.32.27.

      EQUIVALENCE (C(1),TW)                                                      POPOUT    50
C                                                                                POPOUT    51
      DIMENSION OUT(200)                                                         POPOUT    52
C                                                                                POPOUT    53
      EQUIVALENCE (BLK(1),OUT(1))                                                POPOUT    54
      EQUIVALENCE (NTEGER(42),NLINE)                                             POPOUT    55
C                                                                                POPOUT    56
C                                                                                POPOUT    57
      EQUIVALENCE (P(7),CGO)                                                     POPOUT    58
     *,            (P(8),CRO)                                                    POPOUT    59
C                                                                                POPOUT    60
      DIMENSION XNB(12), XEB(12), BIASN(6), BIASE(6), DBIAS(6), XSNC(6)          POPOUT    61
     *,         XTNC(6), XSEC(6), XTEC(6), DXSC(6), DXTC(6), RXEC(6)             POPOUT    62
     *,         DXSM(6), DXTM(6), DRXM(6), SIGEM(18), REM(6,6), RQN(4)           POPOUT    63
     *,         RQE(4), ROM(4)                                                   POPOUT    64
C                                                                                POPOUT    65
      EQUIVALENCE (OUT(1),XNB(1))                                                POPOUT    66
     *,            (OUT(13),XEB(1))                                              POPOUT    67
     *,            (OUT(25),BIASN(1))                                            POPOUT    68
     *,            (OUT(31),BIASE(1))                                            POPOUT    69
     *,            (OUT(37),DBIAS(1))                                            POPOUT    70
     *,            (OUT(43),XSNC(1))                                             POPOUT    71
     *,            (OUT(49),XTNC(1))                                             POPOUT    72
     *,            (OUT(55),XSEC(1))                                             POPOUT    73
     *,            (OUT(61),XTEC(1))                                             POPOUT    74
```

```
     *,                 (OUT(67),DXSC(1))                               POPOUT
     *,                 (OUT(73),DXTC(1))                               POPOUT
     *,                 (OUT(79),RXEC(1))                               POPOUT
     *,                 (OUT(85),DXSM(1))                               POPOUT
     *,                 (OUT(91),DXTM(1))                               POPOUT
     *,                 (OUT(97),DRXM(1))                               POPOUT
     *,                 (OUT(103),SIGEM(1))                             POPOUT
     *,                 (OUT(121),REM(1,1))                             POPOUT
      EQUIVALENCE       (OUT(157),RQN(1))                               POPOUT
     *,                 (OUT(161),RQE(1))                               POPOUT
     *,                 (OUT(165),RQM(1))                               POPOUT
C                          4001                                         POPOUT
C     OUT(1-12)            NAVIGATED BETELGEUSE                         POPOUT
C     OUT(13-24)           ENVIRONMENT BETELGEUSE                       POPOUT
C     OUT(25-30)           NAVIGATED BIASES                             POPOUT
C     OUT(31-36)           ENVIRONMENT BIASES                           POPOUT
C     OUT(37-42)           NAVIGATED MINUS ENVIRONMENT BIASES           POPOUT
C     OUT(43-48)           NAVIGATED CARTESIAN S/C                      POPOUT
C     OUT(49-54)           NAVIGATED CARTESIAN TGT                      POPOUT
C     OUT(55-60)           ENVIRONMENT CARTESIAN S/C                    POPOUT
C     OUT(61-66)           ENVIRONMENT CARTESIAN TGT                    POPOUT
C     OUT(67-72)           NAVIGATED MINUS ENVIRONMENT CARTESIAN S/C    POPOUT
C     OUT(73-78)           NAVIGATED MINUS ENVIRONMENT CARTESIAN TGT    POPOUT
C     OUT(79-84)           TGT MINUS S/C ENVIRONMENT CARTESIAN          POPOUT
C     OUT(85-90)           NAVIGATED MINUS ENVIRONMENT MEASUREMENT S/C  POPOUT
C     OUT(91-96)           NAVIGATED MINUS ENVIRONMENT MEASUREMENT TGT  POPOUT
C     OUT(97-102)          NAVIGATED MINUS ENVIRONMENT RELATIVE STATE   POPOUT
C                          VECTOR, MEASUREMENT                          POPOUT
C     OUT(103-120)         ONE-SIGMA ERRORS MEASUREMENT                 POPOUT
C     OUT(121-156)         RELATIVE COVARIANCE MATRIX MEASUREMENT       POPOUT
```



```
C     OUT(157-160)         RELATIVE QUANTITIES NAVIGATED                POPOUT
C     OUT(161-164)         RELATIVE QUANTITIES ENVIRONMENT              POPOUT
C     OUT(165-168)         RELATIVE QUANTITIES OBSERVED                 POPOUT
C                                                                       POPOUT
      DIMENSION XM(3), YM(3), ZM(3), MFTMAT(3,3), DUM(3,3)              POPOUT
     *,         MFTRN(18,18), WM(18,27), OK(6,18), RWM(6,27)            POPOUT
     *,         RWMT(27,6)
```

```
C
C
      NSKIP = 0                                                    A
      GO TO 1
      ENTRY POPW
      NSKIP = 1
    1 CONTINUE                                                     A
C                                                                  B
C SET UP THE NAVIGATED STATE VECTOR IN THE BETELGEUSE FRAME
      CALL SETUP(Y(2),XNB(1))
C SET UP THE ENVIRONMENT STATE VECTOR IN THE BETELGEUSE FRAME
      CALL SETUP(Y(20),XEB(1))                                     B
C CREATE THE NAVIGATED AND ENVIRONMENT BIAS ARRAYS AND THEIR DIFFERENCE
      DO 5 I=1,6                                                   C
      BIASN(I) = Y(I+13)
      BIASE(I) = Y(I+31)
      DBIAS(I) = BIASN(I) - BIASE(I)
      IF(I.EQ.3.OR.I.EQ.4) DBIAS(I) = DBIAS(I)*1000.0              C
    5 CONTINUE
C CREATE THE NAVIGATED STATE VECTOR IN THE CARTESIAN FRAME
      DO 10 I=1,6
      XSNC(I) = Y(I+37)
      XTNC(I) = Y(I+43)
   10 CONTINUE
C ROTATE ENVIRONMENT STATE VECTOR FROM THE BETELGEUSE TO CARTESIAN FRAME
      CALL CART1(OUT(55),Y(20))                                    D
C CALCULATE THE ERROR VECTOR BETWEEN THE NAVIGATED AND ENVIRONMENT STATE
C VECTORS IN THE CARTESIAN FRAME
      DO 15 I=1,6
      DXSC(I) = XSNC(I) - XSEC(I)
      DXTC(I) = XTNC(I) - XTEC(I)
   15 CONTINUE
C IN THE CARTESIAN FRAME CALCULATE THE RELATIVE STATE VECTOR BETWEEN THE
C ENVIRONMENT STATES OF THE S/C AND TGT
      DO 20 I=1,6
      RXEC(I) = XTEC(I) - XSEC(I)
   20 CONTINUE
C
C*******************************************************************E**********
C                                                                  E
C CREATE THE TRANSFORMATION MATRIX WHICH ROTATES A VECTOR FROM THE
C CARTESIAN TO THE MEASUREMENT FRAME
C
C         YM = UNIT(RANGE)
C         ZM = UNIT(RS X RANGE)
C         XM = UNIT(YM X ZM)
C
      CALL UVEC(RXEC(1),RXEC(2),RXEC(3),XM)
```

```
      CALL UCROSS(XSEC(1),YM,ZM)
      CALL UCROSS(YM,ZM,XM)
      CALL TRN(XM,YM,ZM,MFTMAT,DUM)                                 E
C                                                                   F
C CREATE THE MATRIX MFTRN WHICH ROTATES THE W MATRIX FROM THE CARTESIAN
C TO THE MEASUREMENT FRAME
C
C ZERO THE LOCATIONS FOR THE MATRIX MFTRN
      DO 25 I=1,18
      DO 25 J=1,18
      MFTRN(I,J) = 0.0
   25 CONTINUE
C SET UP MFTRN AS AN (18 X 18) MATRIX COMPOSED OF 4 (3 X 3) MFTMATS AND
C 2 (3 X 3) IDENTITY MATRICES ALONG THE DIAGONAL
      DO 30 I=3,12,3
      L = I - 3
      DO 30 J=1,3
      DO 30 K=1,3
      MFTRN((J+L),(K+L)) = MFTMAT(J,K)
   30 CONTINUE
      DO 35 I=13,18
      MFTRN(I,I) = 1.0
   35 CONTINUE                                                      F
C ROTATE THE W MATRIX INTO THE MEASUREMENT FRAME                    G
      CALL MATMUL(MFTRN,WE,WM,18,18,27)                             G
C CALCULATE THE ONE-SIGMA POSITION AND VELOCITY ERRORS OF THE COVARIANCE
C MATRIX E IN THE MEASUREMENT FRAME                                 H
      DO 40 I=1,18
      ED = 0.0
      DO 40 J=1,27
      ED = ED + WM(I,J)**2
      IF(J.EQ.27) SIGEM(I) = SQRT(ED)
   40 CONTINUE                                                      H
C                                                                   I
C CREATE THE OK MATRIX TO BE USED IN CALCULATING THE RELATIVE W MATRIX
C IN THE MEASUREMENT FRAME
      DO 45 I=1,6
      DO 50 J=1,18
      OK(I,J) = 0.0
   50 CONTINUE
      OK(I,I) = 1.0
      OK(I,(I+6)) = -1.0
   45 CONTINUE                                                      I
```

```
C                                                                       POPOUT
C CREATE THE RELATIVE W MATRIX IN THE MEASUREMENT FRAME             J   POPOUT
      CALL MATMUL(OK,WM,RWM,6,18,27)                                    POPOUT
C CREATE THE RELATIVE COVARIANCE MATRIX IN THE MEASUREMENT FRAME        POPOUT
      CALL MATRAN(RWM,6,27,RWMT)                                        POPOUT
      CALL MATMUL(RWM,RWMT,REM,6,27,6)                              J   POPOUT
C REPLACE THE VARIANCES ALONG THE DIAGONAL OF THE RELATIVE COVARIANCE   POPOUT
C MATRIX WITH THEIR ONE-SIGMAS                                          POPOUT
      DO 55 I=1,6                                                       POPOUT
      REM(I,I) = SQRT(ABS(REM(I,I)))                                    POPOUT
   55 CONTINUE                                                          POPOUT
C CALCULATE THE CORRELATION COEFFICIENTS AND PLACE THEM IN THE LOWER    POPOUT
```



```
C TRIANGLE OF THE SYMMETRIC COVARIANCE MATRIX                           POPOUT
      DO 60 I=1,6                                                       POPOUT
      IF(REM(I,I).EQ.0.0) GO TO 60                                      POPOUT
      DO 59 J=1,6                                                       POPOUT
      IF(J.GE.I) GO TO 59                                               POPOUT
      REM(I,J) = REM(J,I)/(REM(I,I)*REM(J,J))                           POPOUT
   59 CONTINUE                                                          POPOUT
   60 CONTINUE                                                          POPOUT
C                                                                   K   POPOUT
C ROTATE THE ERROR VECTOR BETWEEN THE NAVIGATED AND ENVIRONMENT         POPOUT
C TRAJECTORIES FROM THE CARTESIAN TO THE MEASUREMENT FRAME              POPOUT
      CALL MATMUL(MFTRN,OUT(67),OUT(85),18,18,1)                        POPOUT
C IN THE MEASUREMENT FRAME, CALCULATE THE DIFFERENCE BETWEEN THE        POPOUT
C RELATIVE STATE VECTORS OF THE NAVIGATED AND ENVIRONMENT TRAJECTORIES  POPOUT
      DO 61 I=1,6                                                       POPOUT
      DRXM(I) = DXTM(I) - DXSM(I)                                       POPOUT
   61 CONTINUE                                                      K   POPOUT
C                                                                       POPOUT
C*********************************************************************** POPOUT
C                                                                   L   POPOUT
C DEFINE THE CURRENT NAVIGATION BASE ATTITUDE FOR THE AZIMUTH AND       POPOUT
C ELEVATION ANGLE COMPUTATION                                           POPOUT
      CALL PEF(OUT(43))                                                 POPOUT
C COMPUTE THE ENVIRONMENT RELATIVE QUANTITIES                           POPOUT
      CALL REL(OUT(55),ROE,XNBE,YNBE,ZNBE)                              POPOUT
```

```
C ADD NOISE TO ENVIRONMENT OBSERVABLES                                  POPOUT
      CALL GARBAGE(RQE)                                                 POPOUT
        CALL REL(OUT(55),RQE,XLVE,YLVE,ZLV                              NOSHIT
C COMPUTE THE NAVIGATED RELATIVE QUANTITIES                             POPOUT
        CALL REL(OUT(43),RQN,XLVN,YLVN,ZLVN)                            NOSHIT
      RADEG = 57.2957795                                                POPOUT
      DO 65 I=1,4                                                       POPOUT
C ADD BIASES TO NAVIGATED RELATIVE QUANTITIES                           POPOUT
      RQN(I) = RQN(I) + BIASN(I)                                        POPOUT
C SET UP THE MEASURED RELATIVE QUANTITIES FOR OUTPUT                    POPOUT
      RQM(I) = QQ(I)                                                    POPOUT
C CONVERT AXIMUTH AND ELEVATION ANGLES TO DEGREES                       POPOUT
      IF(I.LT.3) GO TO 65                                               POPOUT
      RQM(I) = RQM(I)*RADEG                                             POPOUT
      RQN(I) = RQN(I)*RADEG                                             POPOUT
      RQE(I) = RQE(I)*RADEG                                             POPOUT
   65 CONTINUE                                                   L      POPOUT
C OUTPUT ONE-SIGMA ERRORS IN THE AZIMUTH AND ELEVATION ANGLES AS        POPOUT
C MILLERADIANS                                                   M      POPOUT
      SIGEM(15) = SIGEM(15)*1000.                                       POPOUT
      SIGEM(16) = SIGEM(16)*1000.                                M      POPOUT
C                                                                N      POPOUT
************************************************************************POPOUT
C                                                                       POPOUT
C CALCULATE THE PRODUCT OF THE UNIVERSAL GRAVITY CONSTANT WITH THE MASS POPOUT
C OF THE EARTH                                                          POPOUT
      GM = CGO*CRO**2                                                   POPOUT
C ADVANCE ESTIMATED TARGET TO ENVIRONMENT TARGET                        POPOUT
      CALL OPEN1(XTNC,XTEC,STT)                                         POPOUT
      CALL OPEN2(XTNC,GM,-STT,RTT,RDTT,VHTT)                            POPOUT
```



```
C                                                                       POPOUT
C ADVANCE ESTIMATED SPACECRAFT TO ENVIRONMENT TARGET                     POPOUT
      CALL OPEN1(XSNC,XTEC,STS)                                         POPOUT
      CALL OPEN2(XSNC,GM,-STS,RTS,RDTS,VHTS)                            POPOUT
C                                                                       POPOUT
C ADVANCE ENVIRONMENT SPACECRAFT TO ENVIRONMENT TARGET                   POPOUT
      CALL OPEN1(XSEC,XTEC,STSA)                                        POPOUT
```

```
      CALL OPEN2(XSEC,GM,-STSA,RTSA,RDTSA,VHTSA)
C COMPUTE CURVILINEAR  ERRORS
      RTE = RSS(XTEC(1),XTEC(2),XTEC(3))
      RDTE = DOT(XTEC(1),XTEC(4))/RTE
      VTE = RSS(XTEC(4),XTEC(5),XTEC(6))
      VHTE = SQRT(VTE**2 - RDTE**2)
C
      SE = (STSA - STS + STT)*RTE
      RE = (RTE - RTSA) - (RTT - RTS)
      VHE = (VHTE - VHTSA) - (VHTT - VHTS)
      RDE = (RDTE - RDTSA) - (RDTT - RDTS)
      PHID = VHTE/RTE
      DVDH = 1.5*PHID*RE
      VCE = VHE + DVDH                                    N
      IF(NSKIP.EQ.1) GO TO 9                              O
C
C     CHECK IF SUFFICIENT TIME HAS ELAPSED FOR A PRINT
      TLP=Y(1) - P(10)
      IF(TLP.LT.P(9)) GO TO 200
      P(10)=Y(1)
C
    9 CONTINUE
C     CHECK FOR TIME TO PAGEHEAD
      IF(NLINE.LT.58) GO TO 11
      PRINT 95
      NLINE=0
   11 CONTINUE
C
**********************************************************************
C
C                        PRINT OUT THE GOODIES
C
      IF(NLINE.EQ.29) PRINT 90
C OUTPUT THE NAVIGATED AND ENVIRONMENT BETELGEUSE STATES
      PRINT 100, Y(1)
      PRINT 105
      PRINT 110, (XNB(I),I=1,12), (XEB(I),I=1,12)
C OUTPUT THE NAVIGATED AND ENVIRONMENT CARTESIAN STATES
      PRINT 115, Y(1)
      PRINT 120
      PRINT 125, (XSNC(I),I=1,6), (XTNC(I),I=1,6)
     *,         (XSEC(I),I=1,6), (XTEC(I),I=1,6)
C OUTPUT CARTESIAN STATE ERRORS IN THE MEASUREMENT FRAME
      PRINT 130
      PRINT 135
      PRINT 140, (DXSM(I),I=1,6), (DXTM(I),I=1,6)
C OUTPUT RELATIVE STATE ERRORS, RELATIVE PARAMETERS, MARKS, COVARIANCE
C OF RELATIVE ERRORS IN THE MEASUREMENT FRAME, AND INERTIAL STANDARD
```

```
C DEVIATIONS
      PRINT 145
      PRINT 150, (DBIAS(I),I=1,6), (DRXM(I),I=1,6)
      PRINT 155
      PRINT 160, SE, VHE, VCE, RE, RDE
      PRINT 165, ICOMP, ZTZ(1)
     *,          (RQN(I),I=1,4), NS(1), ZTZ(2)
     *,          (RQE(I),I=1,4), NS(2), ZTZ(3), (REM(1,I),I=1,6)
     *,          (RQM(I),I=1,4), NS(3), ZTZ(4), (REM(2,I),I=1,6)
     *,     meas                                (REM(3,I),I=1,6)
                       R, Ṙ, AZ
     *,                                  SZ(1), (REM(4,I),I=1,6)
     *,          (SIGEM(I),I=1,6),       SZ(2), (REM(5,I),I=1,6)
     *,          (SIGEM(I),I=7,12),      SZ(3), (REM(6,I),I=1,6)
     *,          (SIGEM(I),I=13,18),     SZ(4)
      NLINE = NLINE + 29
C
************************************************************************
C
C                        FORMAT STATEMENTS
C
   90 FORMAT(//)
   95 FORMAT(1H1)
  100 FORMAT(53X,17HBETELGEUSE STATE ,F8.1)
  105 FORMAT(12X,117HRAD VEC  LONGITUDE LATITUDE ALT RATE  HOR VEL   HEA
     *DING     X-BAR      Y-BAR      Z-BAR      U-BAR      V-BAR      W-BA
     *R)
  110 FORMAT(1X,10HNAV       ,F9.0,2F9.3,F9.2,F10.2,F9.3,3F11.0,3F10.2,
     *      /1X,10HACT       ,F9.0,2F9.3,F9.2,F10.2,F9.3,3F11.0,3F10.2)
  115 FORMAT(/53X,17HCARTESIAN STATE  ,F8.1)
  120 FORMAT(7X,123HXS(BRF)     YS(BRF)     ZS(BRF)    XSD(BRF)  YSD(BRF)
     * ZSD(BRF)   XT(BRF)     YT(BRF)     ZT(BRF)    XTD(BRF)   YTD(BRF)   ZT
     *D(BRF))
  125 FORMAT(1X,3HNAV,3F11.0,3F10.2,3F11.0,3F10.2/
     *        1X,3HACT,3F11.0,3F10.2,3F11.0,3F10.2)
  130 FORMAT(/42X,43HCARTESIAN STATE ERRORS IN MEASUREMENT FRAME)
  135 FORMAT(7X,123HXSE(MF)     YSE(MF)     ZSE(MF)    XSDE(MF)  YSDE(MF)
     * ZSDE(MF)   XTE(MF)     YTE(MF)     ZTE(MF)    XTDE(MF)   YTDE(MF)   ZT
     *DE(MF))
  140 FORMAT(4X,3F11.0,3F10.2,3F11.0,3F10.2)
  145 FORMAT(/56X,21HRELATIVE STATE ERRORS,/6X,124H====================
     *=======BIAS ESTIMATION======================******************POSITI
     *ON AND VELOCITY************************/10X,120HRANGE      R-RATE
```

```
     *(MF)     XRDE(MF)    YRDE(MF)    ZRDE(MF))                          POPOUT    367
 150  FORMAT(6X,F9.2,2F11.3,3F10.3,3F11.0,3F10.2)                         POPOUT    368
  155 FORMAT(74X,49HD-R        S-DOT     SD-       DELTA-H  DELH-DOT    ) POPOUT    3
 160  FORMAT(16X,52HRELATIVE PARAMETERS                MARKS              POPOUT    3
     * ,F10.0,F10.3,F11.3,F11.0,F10.3)                                    POPOUT    371
 165  FORMAT(10X,40HRANGE       R-RATE   AZIMUTH REL ELEV  MFLG,I2,6H  UR POPOUT    372
     * F6.1/1X,6HNAV    ,F9.0,F10.2,2F9.3,6H  RAD ,I2,6H  URD ,F6.1,13X,  POPOUT    373
     * 36HCOVARIANCE OF RELATIVE ERRORS (MF)   ,/1X,6HACT    ,F9.0,F10.2, POPOUT    374
     * 2F9.3,6H  VHF ,I2,6H  UAZ ,F6.1,3H  X,6E10.3,/1X,6HMEAS. ,F9.0,    POPOUT    375
     *F10.2,2F9.3,5H  OPT,I3,6H  UEL ,F6.1,3H  Y,F10.5,  5E10.3,/64X,3H   POPOUT    376
     * Z,2F10.5,         4E10.3,/8X,50HINERTIAL STANDARD DEVIATIONS       POPOUT    377
     *               SR  ,F5.3,4H  XD, 3F10.5,              3E10.3/1X,4HS/C POPOUT  378
```

UBROUTINE POPOUT   CDC 6600 FTN V3.0-P308 OPT=1 08/29/72 11.32.27.

```
     *,3F8.0,3F7.2,8H    SRD ,F5.3,4H  YD, 4F10.5,              2E10.3,/  POPOUT    379
     *1X,4HTGT ,3F8.0,3F7.2,8H    SAZ ,F5.3,4H  ZD, 5F10.5,               POPOUT    380
     *E10.3,/1X,4HBIAS,F8.0,2F8.3,3F7.2,8H    SEL ,F5.3)                  POPOUT    381
 200  CONTINUE                                                            POPOUT    382
           RETURN                                                         POPOUT    383
           END                                                            POPOUT    384
```

CART

ROUTINE CART1

ENTRY CART1 CONVERTS A STANDARD BETELGEUSE VECTOR (S/C AND TGT) TO A STANDARD CARTESIAN VECTOR IN THE BRF FRAME (NORTH POINTING, EARTH CENTERED, INERTIAL). AS USUAL, BETELGEUSE LONGITUDE IS A POSITIVE ROTATION ABOUT THE SOUTH POLE.

ENTRY CART2 IS THE FUNCTIONAL INVERSE OF CART1.

```
            SUBROUTINE CART1(X,Y)
C
            DIMENSION X(12),Y(12)
C
C     SUBROUTINE TO CONVERT A BETEL VECTOR TO PLANET CENTERED INERTIAL
C     Y=BETELGEUSE                                    X=CARTESIAN
      CMU=COS(Y(2))
      SMU=SIN(Y(2))
      CLA=COS(Y(3))
      SLA=SIN(Y(3))
C
      X(1)=  Y(1)*CLA*CMU
      X(2)= -Y(1)*CLA*SMU
      X(3)=  Y(1)*SLA
      X(4)=  Y(7)*CLA*CMU-Y(8)*SMU+Y(9)*SLA*CMU
      X(5)= -Y(7)*CLA*SMU-Y(8)*CMU-Y(9)*SLA*SMU
      X(6)=  Y(7)*SLA-Y(9)*CLA
      X(7)=  (Y(1)+Y(4))*CLA*CMU-Y(5)*SMU+Y(6)*SLA*CMU
      X(8)= -(Y(1)+Y(4))*CLA*SMU-Y(5)*CMU-Y(6)*SLA*SMU
      X(9)=  (Y(1)+Y(4))*SLA-Y(6)*CLA
      X(10)= (Y(7)+Y(10))*CLA*CMU-(Y(8)+Y(11))*SMU+(Y(9)+Y(12))*SLA*CMU
      X(11)=-(Y(7)+Y(10))*CLA*SMU-(Y(8)+Y(11))*CMU-(Y(9)+Y(12))*SLA*SMU
      X(12)= (Y(7)+Y(10))*SLA-(Y(9)+Y(12))*CLA
      RETURN
      ENTRY CART2
C     SUBROUTINE TO CONVERT PLANET CENTERED INERTIAL TO A BETEL VECTOR
C
      Y(1)=RSS(X(1),X(2),X(3))
      Y(2)=-ATAN2(X(2),X(1))
      Y(3)=ASIN(X(3)/Y(1))
      CMU=COS(Y(2))
      SMU=SIN(Y(2))
      CLA=COS(Y(3))
      SLA=SIN(Y(3))
      Y(4)= (X(7)-X(1))*CLA*CMU-(X(8)-X(2))*CLA*SMU+(X(9)-X(3))*SLA
      Y(5)=-(X(7)-X(1))*SMU-(X(8)-X(2))*CMU
      Y(6)= (X(7)-X(1))*SLA*CMU-(X(8)-X(2))*SLA*SMU-(X(9)-X(3))*CLA
      Y(7)= X(4)*CLA*CMU-X(5)*CLA*SMU+X(6)*SLA
      Y(8)=-X(4)*SMU-X(5)*CMU
      Y(9)= X(4)*SLA*CMU-X(5)*SLA*SMU-X(6)*CLA
      Y(10)= (X(10)-X(4))*CLA*CMU-(X(11)-X(5))*CLA*SMU+(X(12)-X(6))*SLA
      Y(11)=-(X(10)-X(4))*SMU-(X(11)-X(5))*CMU
      Y(12)= (X(10)-X(4))*SLA*CMU-(X(11)-X(5))*SLA*SMU-(X(12)-X(6))*CLA
      RETURN
      END
```

SETY

```
ATAN2       REAL        2 LIBRARY       29
COS         REAL        1 LIBRARY        7          9          31          33
RSS         REAL        3               28
SIN         REAL        1 LIBRARY        8         10          32          34

ICS
AM LENGTH      430B       280
```

```
SUBROUTINE  SETY                          CDC 6600 FTN V3.0-P308 OPT=1  08/29/72  11.32.27.

                  SUBROUTINE SETY(Y)                                      SETY          2
      C                                                                   SETY          3
      C         SUBROUTINE TO COMPUTE  STATE VECTORS OF INTEREST-        SETY          4
      C         Y(1)=TIME                                                 SETY          5
      C         Y(2)-Y(19)  NAVIGATED BETELGEUSE STATE                    SETY          6
      C         Y(20)-Y(37) ENVIRONMENT BETELGEUSE STATE                  SETY          7
      C         Y(38)-Y(55) NAVIGATED CARTESIAN STATE                     SETY          8
      C         Y(56)-Y(67) L.T. S/C ERROR STATE                          SETY          9
      C         Y(68)-Y(79) L.T. TGT ERROR STATE                          SETY         10
      C         Y(80)-Y(97) CARTESIAN DEVIATION VECTOR                    SETY         11
      C                                                                   SETY         12
                  DIMENSION  Y(100),XE(12),XCS(12),XCT(12),RXVT(3,3)      SETY         13
       1          ,XS(12),XT(12)
```

ROUTINE SETY

THIS ROUTINE IS LEFT OVER FROM AN EARLY VERSION OF THE NAVIGATION PROGRAM. IT HAS ONLY ONE OPERATIVE INSTRUCTION AND CALLS TO SETY MAY BE REPLACED BY THAT INSTRUCTION WHEREVER THEY ARE FOUND IN THE PROGRAM.

AREA A — CALL TO CART1 IS ONLY OPERATIVE INSTRUCTION IN ROUTINE. LOADS BETELGEUSE ESTIMATED STATE INTO Y(38)-Y(49) AS CARTESIAN VECTOR.

```
C
C
      CONSTRUCT CARTESIAN VECTORS FOR NAV AND ENV STATES          A
            CALL CART1(Y(38),Y(2))
      RETURN                                                      A
      ENTRY FAKE
            CALL CART1(XE,Y(20))
C     DEFINE CARTESIAN DIFFERENCE VECTOR
            CALL MATADD(XE,Y(38),Y(80),12,1,2)
C     LOAD UPPER CART. DEV. VECTOR WITH ESTIMATED CONSTANTS
C     CONSTRUCT PSEUDO-BETELGEUSE VECTORS FOR NAV AND ENV
            DO 10 I=1,6
            Y(I+91)=Y(I+31)-Y(I+13)
            XS(I)=Y(I+37)
            XS(I+6)=XE(I)
            XT(I)=Y(I+43)
            XT(I+6)=XE(I+6)
   10       CONTINUE
            CALL CART2(XS,XCS)
            CALL CART2(XT,XCT)
C           CONSTRUCT TRANSFORM TO LOCAL TANGENT FOR S/C
            CALL TBG(XCS(7),XCS(8),XCS(9),RXVT)
            CALL MATMUL(RXVT,XCS( 4),Y(62),3,3,1)
            CALL MATMUL(RXVT,XCS( 7),Y(59),3,3,1)
            CALL MATMUL(RXVT,XCS(10),Y(65),3,3,1)
C     CONSTRUCT TRANSFORM TO LOCAL TANGENT FOR TGT
            CALL TBG(XCT(7),XCT(8),XCT(9),RXVT)
            CALL MATMUL(RXVT,XCT( 4),Y(68),3,3,1)
            CALL MATMUL(RXVT,XCT( 7),Y(77),3,3,1)
            CALL MATMUL(RXVT,XCT(10),Y(71),3,3,1)
            DO 30 I=1,3
            Y(I+55)=XCS(I)
            Y(I+73)=XCT(I)
   30       CONTINUE
            RETURN
            END
```

REL

ROUTINE REL

SUBROUTINE REL COMPUTES RANGE, R-RATE, AZIMUTH AND ELEVATION FROM A TWO-VEHICLE CARTESIAN VECTOR AND THREE ORTHONORMAL VECTORS. CONTRARY TO COMMENT CARD, UNIT VECTORS NEED NOT BE NAV BASE VECTORS (SEE AREA L, ROUTINE POPOUT).

```
      SUBROUTINE REL(Y,S,REF1,REF2,REF3)
C
      DIMENSION Y(12), S(4)
C
      COMMON VAR
      DIMENSION VAR(5600), P(5000), SAVE(950), BLK(700)
      EQUIVALENCE (VAR(601),P(1))
      EQUIVALENCE (P(350),SAVE(1))
     *,           (P(1300),BLK(1))
      DIMENSION REF1(3), REF2(3), REF3(3)
C
      DIMENSION DX(6),DXNB(6),UR(3)
C
C     SUBROUTINE TO COMPUTE RANGE, RANGE RATE, AZIMUTH AND ELEVATION IN
C     CURRENT NAVIGATION BASE COORDINATES
C
      DO 5 I=1,6
    5 DX(I)=Y(I+6)-Y(I)
      CALL UVEC(DX(1),DX(2),DX(3),UR)
      S(1)=DOT(UR,DX(1))
      S(2)=DOT(UR,DX(4))
      S(3)=ATAN(DOT(UR,REF3)/DOT(UR,REF2))
      S(4)=ASIN(DOT(UR,REF1))
      RETURN
      END
```

SETUP

ROUTINE SETUP

SETS UP OUTPUT FORM OF BETELGEUSE STATE.
CONVERTS LONGITUDE AND LATITUDE TO DEGREES.
COMPUTES S/C HORIZONTAL SPEED.
DEFINES HEADING ANGLE AS A POSITIVE ROTATION ABOUT S/C RADIUS VECTOR FROM DUE WEST.

```
      SUBROUTINE SETUP(Y,OUT)
C
C     SETS UP A STANDARD BETEL OUTPUT
C
      DIMENSION Y(12),OUT(12)
C
      OUT( 1)=Y(1)
      OUT( 2)=Y(2)*57.2957795
      OUT( 3)=Y(3)*57.2957795
      OUT( 4)=Y(7)
      OUT(5)=RSS(Y(8),Y(9),0.)
      OUT(6)=ATAN2(Y(9),Y(8))*57.2957795
      DO 5 I=1,3
      OUT(I+6)=Y(I+3)
    5 OUT(I+9)=Y(I+9)
      RETURN
      END
```

GARBAGE

ROUTINE GARBAGE

SENSOR NOISE MODEL. CALCULATES BIASES AND NOISE FROM A A PREUDO-RANDOM NORMAL DISTRIBUTION. LOADS RELATIVE PARAMETER MEASURED VALUE LOCATION FOR BVEC.

AREA A — CHECK TO SEE IF THIS IS FIRST PASS THROUGH THIS ROUTINE ON THIS MONTE-CARLO CYCLE. IF NOT, GO TO NOISE COMPUTATIONS. OTHERWISE, COMPUTE VALUES FOR RANGE, R-RATE, AZ AND EL BIASES.
LOAD BIASES INTO ACTUAL BETELGEUSE STATE, Y(32)-Y(35).

AREA B — STANDARD DEVIATION COMPUTATIONS. FOR RANGE AND R-RATE, SIGMA IS A FRACTION OF THE PARAMETER VALUE, DOWN TO A MINIMUM. AREA B SELECTS MAX OF PARAMETER FRACTION AND ITS MINIMUM. THIS COMPUTATION IS PERFORMED EACH PASS THROUGH GARBAGE.

AREA C — THIS INSTRUCTION ASSURES THAT THE RANGE MEASUREMENT IS A POSITIVE NUMBER. AT SMALL RANGES, QQ(1) COULD BECOME NEGATIVE AFTER THE ADDITION OF NOISE AND BIASES TO S(1).

```
      SUBROUTINE GARBAGE(S)
C
      DIMENSION  S(4)
C
      COMMON VAR
      DIMENSION VAR(5600), NTEGER(100), P(5000), SAVE(950), BLK(700)
     *,         Y(100)
      EQUIVALENCE (VAR(1),Y(1))
     *,           (VAR(401),NTEGER(1))
     *,           (VAR(601),P(1))
      EQUIVALENCE (NTEGER(29),NGUIDE)
      EQUIVALENCE (P(350),SAVE(1))
     *,           (P(1300),BLK(1))
      DIMENSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)
     *,         ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)
     *,         USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)
     *,         SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)
     *,         ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)
     *,         X(18), WE(18,27)
      EQUIVALENCE (SAVE(1),QQ(1)),          (SAVE(5),SIG(1))
     *,           (SAVE(9),C(1)),           (SAVE(19),REFMAT(1,1))
     *,           (SAVE(28),XNBN(1)),       (SAVE(31),YNBN(1))
     *,           (SAVE(34),ZNBN(1)),       (SAVE(37),NE(1))
     *,           (SAVE(47),NM(1)),         (SAVE(57),DTL(1))
     *,           (SAVE(67),DTN(1)),        (SAVE(77),DTM(1))
     *,           (SAVE(87),USP(1)),        (SAVE(97),USV(1))
     *,           (SAVE(107),UTP(1)),       (SAVE(117),UTV(1))
     *,           (SAVE(127),SR(1)),        (SAVE(137),SRD(1))
     *,           (SAVE(147),SO(1)),        (SAVE(157),SC1(1))
     *,           (SAVE(167),SC2(1)),       (SAVE(177),NW(1))
     *,           (SAVE(187),TLM(1)),       (SAVE(197),NS(1))
     *,           (SAVE(200),ZTZ(1)),       (SAVE(204),SZ(1))
     *,           (SAVE(208),TALIGN(1)),    (SAVE(218),NALIGN)
     *,           (SAVE(229),XNBE(1)),      (SAVE(232),YNBE(1))
     *,           (SAVE(235),ZNBE(1)),      (SAVE(258),X(1))
     *,           (SAVE(276),WE(1,1))
C
      EQUIVALENCE (C(1),TW)
C
C
      COMMON/GARB/RVAR,RVARMIN,VVAR,VVARMIN,VARAZ,VAREL,NFAMV
     *,           BR,BV,BAZ,BEL,NFAMB
```

```
C
C        CHECK IF INITIAL VALUES FO BIASES HAVE BEEN SET IN          A
         IF(VAR(1).GT.1.) GO TO 5
         BR=UNURN(0,NFAMB,0.,BRO)
         BV=UNURN(0,NFAMB,0.,BVO)
         BAZ=UNURN(0,NFAMB,0.,BAZO)
         BEL=UNURN(0,NFAMB,0.,BELO)
         Y(32) = BR
         Y(33) = BV
         Y(34) = BAZ
         Y(35) = BEL                                                 A
    5    CONTINUE
```



```
C        COMPUTE MEASUREMENT NOISE AND ADD BIASES                    B
         SIG(1)=AMAX1(S(1)*RVAR,RVARMIN)
         SIG(2)=AMAX1(ABS(S(2)*VVAR),VVARMIN)
         SIG(3)=VARAZ
         SIG(4)=VAREL                                                B
         QQ(1) = S(1) + UNURN(0,NFAMV,BR,SIG(1))
         QQ(1) = ABS(QQ(1))                                          C
         QQ(2) = S(2) + UNURN(0,NFAMV,BV,SIG(2))
         QQ(3) = S(3) + UNURN(0,NFAMV,BAZ,SIG(3))
         QQ(4) = S(4) + UNURN(0,NFAMV,BEL,SIG(4))
         IF(VAR(1).LT.5.) PRINT 100,BR,BV,BAZ,BEL,(SIG(I),I=1,4)
  100    FORMAT(/1X,8E15.6)
         RETURN
         END
```

ALIGN

ROUTINE ALIGN

SIMPLE-MINDED PLATFORM MODEL. CREATES DRIFT RATES AT INITIAL PASS, MISALIGNMENT BIASES EVERY ALIGNMENT.

AREA A CHECK THE ALIGNMENT COUNTER TO SEE IF THIS PASS IS FIRST ALIGNMENT. IF IT IS VISIT UNURN TO CREATE INITIAL PLATFORM DRIFT RATES. OTHERWISE, PASS TO MISALIGNMENT COMPUTATION.

AREA B INCREMENT THE ALIGNMENT COUNTER.

AREA C CALCULATE THE STARTING LOCATION IN DATA ARRAY FOR DATA FROM THIS ALIGNMENT.
STORE THREE RATES AND THREE INITIAL MISALIGNMENTS.

AREA D REFER TOTAL PLATFORM DRIFT TO TIME OF ALIGNMENT. ON INPUT, THE FIRST ALIGNMENT MAY BE SPECIFIED AS HAVING OCCURRED AT A TIME PREVIOUS TO THE START OF THE RUN: FOR EXAMPLE, TALIGN(1) = -600.SPECIFIES AN ALIGNMENT 10 MINUTES BEFORE START OF RUN. AREA D CALCULATES TOTAL DRIFT TO PRESENT TIME.

AREA E CONVERT DRIFT RATE FROM RAD/SEC TO MR/HR AND INITIAL MISALIGNMENT FROM RAD TO MR FOR OUTPUT.

AREA F STORE TIME OF ALIGNMENT.

AREA G COMPUT REFMAT. THIS IS DEFINED AS THE ESTIMATED TRANSFORMATION MATRIX FROM BRF TO PLATFORM AXES. APOLLO USAGE DEFINES "NOMINAL" ALIGNMENT AS PLATFORM AXES COINCIDENT WITH LOCAL VERTICAL UNIT VECTORS AT TALIGN.

```
      SUBROUTINE ALIGN
C
C     SUBROUTINE TO SIMULATE THE PERFORMANCE OF A PLATFORM ALIGNMENT
C
      COMMON VAR
      DIMENSION VAR(5600), Y(100), DYDX(100), Q(100), FIRSTY(100)
     *,          NTEGER(100), D(100), P(5000)
      EQUIVALENCE (VAR(1),Y(1))
     *,            (VAR(101),DYDX(1))
     *,            (VAR(201),Q(1))
     *,            (VAR(301),FIRSTY(1))
     *,            (VAR(401),NTEGER(1))
     *,            (VAR(501),D(1))
     *,            (VAR(601),P(1))
      DIMENSION SAVE(950), BLK(700), DATA(350), COV(24,24)
      EQUIVALENCE (P(350),SAVE(1))
     *,            (P(1300),BLK(1))
     *,            (P(4074),DATA(1))
     *,            (P(4424),COV(1,1))
      DIMENSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)
     *,          ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)
     *,          USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)
     *,          SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)
     *,          ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)
     *,          X(18), WE(18,27)
      EQUIVALENCE (SAVE(1),QQ(1)),          (SAVE(5),SIG(1))
     *,            (SAVE(9),C(1)),           (SAVE(19),REFMAT(1,1))
     *,            (SAVE(28),XNBN(1)),       (SAVE(31),YNBN(1))
     *,            (SAVE(34),ZNBN(1)),       (SAVE(37),NE(1))
     *,            (SAVE(47),NM(1)),         (SAVE(57),DTL(1))
     *,            (SAVE(67),DTN(1)),        (SAVE(77),DTM(1))
     *,            (SAVE(87),USP(1)),        (SAVE(97),USV(1))
     *,            (SAVE(107),UTP(1)),       (SAVE(117),UTV(1))
     *,            (SAVE(127),SR(1)),        (SAVE(137),SRD(1))
     *,            (SAVE(147),SO(1)),        (SAVE(157),SC1(1))
     *,            (SAVE(167),SC2(1)),       (SAVE(177),NW(1))
     *,            (SAVE(187),TLM(1)),       (SAVE(197),NS(1))
     *,            (SAVE(200),ZTZ(1)),       (SAVE(204),SZ(1))
     *,            (SAVE(208),TALIGN(1)),    (SAVE(218),NALIGN)
     *,            (SAVE(229),XNBE(1)),      (SAVE(232),YNBE(1))
     *,            (SAVE(235),ZNBE(1)),      (SAVE(258),X(1))
     *,            (SAVE(276),WE(1,1))
```

```
     *         DUMMY(3,3)                                                 ALIGN   45
      EQUIVALENCE (Y(98),DRIFT(1))                                        ALIGN   46
     *,           (DYDX(98),RATE(1))                                      ALIGN   47
     *,           (NTEGER(33),LIGN)                                       ALIGN   48
      EQUIVALENCE (BLK(1),RT(1))                                          ALIGN   49
     *,           (BLK(4),DRFT(1))                                        ALIGN   50
     *,           (BLK(7),UX(1))                                          ALIGN   51
     *,           (BLK(10),UY(1))                                         ALIGN   52
     *,           (BLK(13),UZ(1))                                         ALIGN   53
C                                                                         ALIGN   54
C     PLATFORM MODEL IS 3-GIMBAL WITH INITIAL 1-SIGMA BIASES AND DRIFT    ALIGN   55
C     AS DEFINED BELOW                                                    ALIGN   56
```

SUBROUTINE ALIGN   CDC 6600 FTN V3.0-P308 OPT=1   08/29/72   11.32.27.

```
C                                                                         ALIGN   57
      COMMON/ALIG/GDR,ALIGNB,NFAMB                                        ALIGN   58
C                                                                         ALIGN   59
C     CHECK IF THIS IS FIRST ALIGNMENT                        A           ALIGN   60
      IF(LIGN.GT.0) GO TO 10                                              ALIGN   61
C     SET UP INITIAL DRIFT RATES FOR THIS RUN                             ALIGN   62
      DO 5 I=1,3                                                          ALIGN   63
    5 RATE(I)=UNURN(0,NFAMB,0.,GDR)                                       ALIGN   64
   10 CONTINUE                                                A           ALIGN   65
      LIGN=LIGN + 1                                           B           ALIGN   66
C     CONSTRUCT ALIGNMENT BIASES                                          ALIGN   67
      DO 15 I=1,3                                                         ALIGN   68
   15 DRIFT(I)=UNURN(0,NFAMB,0.,ALIGNB)                                   ALIGN   69
C     REFER INTEGRATED DRIFT TO TIME OF ALIGNMENT AND SCAL OUTPUTS        ALIGN   70
      DT=Y(1) - TALIGN(LIGN)                                              ALIGN   71
      DO 20 I=1,3                                             C           ALIGN   72
C     CALCULATE INDEX TO STORE ALIGNMENT DATA IN ARRAY                    ALIGN   73
      MM=(LIGN-1)*7 + 250 + I                                             ALIGN   74
      DATA(MM)=RATE(I)                                                    ALIGN   75
      DATA(MM+3)=DRIFT(I)                                     C           ALIGN   76
      DRIFT(I)=DRIFT(I) + DT*RATE(I)                          D           ALIGN   77
      RT(I)=RATE(I)*3600000.                                  E           ALIGN   78
   20 DRFT(I)=DRIFT(I)*1000.                                  E           ALIGN   79
      MM=LIGN*7+250                                           F           ALIGN   80
      DATA(MM)=TALIGN(LIGN)
```

```
C
C         COMPUTE REFMAT AT TALIGN, DEFINED AS THE NOMINAL-        G
C                    SMX= UNIT(R)
C                    SMY= UNIT(R X V)
C                    SMZ= UNIT(SMX X SMY)
C
          CALL UVEC(Y(38),Y(39),Y(40),UX)
          CALL UCROSS(UX,Y(41),UY)
          CALL UCROSS(UX,UY,UZ)
          CALL TRN(UX,UY,UZ,REFMAT,DUMMY)                          G
          PRINT 100, LIGN, (DRFT(I),I=1,3), (RT(I),I=1,3)
  100     FORMAT(/14X,24HTHIS IS ALIGNMENT NUMBER,I5,23H  BIASES AND RATES
        1ARE-,6F10.5)
          RETURN
          END
```

REF

ROUTINE REF

COMPUTES UNIT VECTORS OF ACTUAL LOCAL VERTICAL, ACTUAL NAV BASE, ESTIMATED LOCAL VERTICAL AND ESTIMATED NAV BASE. ACTUAL AND ESTIMATED QUANTITIES DIFFER BY PLATFORM DRIFT. CURRENT NAV BASE VECTORS ARE DEFINED AS THE LINE-OF-SIGHT SYSTEM. SUBROUTINE IS ALWAYS CALLED WITH ESTIMATED CARTESIAN STATE.

AREA A CALCULATE ESTIMATED LOCAL VERTICAL VECTORS.

AREA A1 CALCULATE ESTIMATED LINE-OF-SIGHT VECTORS.

AREA B CALCULATE THE TOTAL PLATFORM DRIFT MATRIX, GAMD. GAMD IS THE TRANSFORMATION FROM ACTUAL TO ESTIMATED PLATFORM AXES. LET V BE ANY ESTIMATE OF A VECTOR IN THE BRF FRAME AND V* BE ITS VALUE AFTER APPLICATION OF PLATFORM ERRORS:

$$\underline{V}_P = \underline{REFMAT} \times \underline{V} \qquad \text{VECTOR IN ESTIMATED PLATFORM FRAME}$$

$$\underline{V}^*_P = \underline{GAMD}^T \times \underline{V}_P \qquad \text{VECTOR IN ACTUAL PLATFORM FRAME}$$

$$\underline{V}^* = \underline{REFMAT}^T \times \underline{V}^*_P \qquad \text{DISTURBED VECTOR IN BRF FRAME}$$

$$= \underline{REFMAT}^T \times \underline{GAMD}^T \times \underline{REFMAT} \times \underline{V}$$

$$= (\underline{GAMD} \times \underline{REFMAT})^T \times \underline{REFMAT} \times \underline{V}$$

CALCULATE $\underline{GAMD} \times \underline{REFMAT}$.

CALULATE $(\underline{GAMD} \times \underline{REFMAT})^T$.

CALCULATE $(\underline{GAMD} \times \underline{REFMAT})^T \times \underline{REFMAT}$.

AREA C APPLY THE JUST COMPUTED MATRIX TO THE ESTIMATED LOCAL VERTICAL VECTORS TO GET THE ACTUAL VECTORS.
APPLY THE SAME MATRIX TO THE ESTIMATED NAV BASE VECTORS TO GET THE ACTUAL NAV BASE VECTORS.

```
      SUBROUTINE REF(Y)                                                 REF        2
C                                                                       REF        3
C     SUBROUTINE TO COMPUTE THE ACTUAL (XNBE,YNBE,ZNBE) AND ASSUMED     REF        4
C     (XNBN,YNBN,ZNBN) UNIT VECTORS OF THE NAVIGATION BASE AXES IN      REF        5
C     THE BASIC REFERENCE FRAME                                         REF        6
C     SUBROUTINE ALSO COMPUTES UNIT VECTORS OF THE ASSUMED (XLVN,YLVN,  NOSHIT    10
C     ZLVN) AND ACTUAL (XLVE,YLVE,ZLVE) LOCAL VERTICAL IN-PLANE FRAME   NOSHIT    11
C                                                                       REF        7
      DIMENSION Y(12)                                                   REF        8
C                                                                       REF        9
      COMMON VAR                                                        REF       10
      DIMENSION VAR(5600), P(5000), SAVE(950), BLK(700)                 REF       11
      EQUIVALENCE (VAR(601),P(1))                                       REF       12
      EQUIVALENCE (P(350),SAVE(1))                                      REF       13
     *,           (P(1300),BLK(1))                                      REF       14
      DIMENSION REFMAT(3,3), XNBN(3), YNBN(3), ZNBN(3)                  REF       15
      EQUIVALENCE (SAVE(19),REFMAT(1,1))                                REF       16
     *,           (SAVE(28),XNBN(1))                                    REF       17
     *,           (SAVE(31),YNBN(1))                                    REF       18
     *,           (SAVE(34),ZNBN(1))                                    REF       19
      DIMENSION DUM(3,3), DUMT(3,3), DICKUP(3,3), GAMD(3,3), YB(100)    REF       20
     *,         XLVE(3),YLVE(3),ZLVE(3),XLVN(3),YLVN(3),ZLVN(3)         NOSHIT    12
      EQUIVALENCE (P(1),YB(1))                                          REF       21
      EQUIVALENCE (YB(98),D1)                                           REF       22
     *,           (YB(99),D2)                                           REF       23
     *,           (YB(100),D3)                                          REF       24
      EQUIVALENCE (BLK(201),DUM(1,1))                                   REF       25
     *,           (BLK(210),DUMT(1,1))                                  REF       26
     *,           (BLK(219),GAMD(1,1))                                  REF       27
     *,           (BLK(228),DICKUP(1,1))                                REF       28
     *,         (SAVE(762),XLVE(1))                                     NOSHIT    13
     *,         (SAVE(765),YLVE(1))                                     NOSHIT    14
     *,         (SAVE(768),ZLVE(1))                                     NOSHIT    15
     *,         (SAVE(771),XLVN(1))                                     NOSHIT    16
     *,         (SAVE(774),YLVN(1))                                     NOSHIT    17
     *,         (SAVE(777),ZLVN(1))                                     NOSHIT    18
C                                                                       REF       29
C     SUBROUTINE COMPUTES TRANSFORM FROM B.R.F. TO NAV. BASE FRAME      REF       30
C                                                                       REF       31
*     CURRENT NAV BASE IS LINE-OF-SIGHT SYSTEM DEFINED BY               NOSHIT    19
*                                                                       NOSHIT    20
*     UNIT(Y) = UNIT(RT - RS)                                           NOSHIT    21
*     UNIT(X) = UNIT(YNBN X ZLVN)
```

```
*     UNIT(Z) = UNIT(XNBN X YNBN)                                      NOSHIT      23
*                                                                      NOSHIT      24
      CALCULATE LOCAL VERTICAL UNIT VECTO                        A     NOSHIT      25
                                                                       NOSHIT      26
        CALL UVEC(Y(1),Y(2),Y(3),XLVN)                                 NOSHIT      27
        CALL UCROSS(XLVN,Y(4),ZLVN)                                    NOSHIT      28
        CALL UCROSS(ZLVN,XLVN,YLVN)                                    NOSHIT      29
C                                                                A     NOSHIT      30
C       COMPUT NAV BASE UNIT VECTORS                                   NOSHIT      31
C                                                                      NOSHIT      32
        DX1 = Y(7) - Y(1)                                              NOSHIT      33
        DX2 = Y(8) - Y(2)                                              NOSHIT      34
```

SUBROUTINE REF   CDC 6600 FTN V3.0-P308 OPT=1   08/29/72   11.32.27.

```
        DX3 = Y(9) - Y(3)                                              NOSHIT      35
        CALL UVEC(DX1,DX2,DX3,YNBN)                                    NOSHIT      36
        CALL UCROSS(YNBN,ZLVN,XNBN)                                    NOSHIT      37
        CALL UCROSS(XNBN,YNBN,ZNBN)                                    NOSHIT      38
C                                                                      NOSHIT      39
C                                                                      REF         42
C       COMPUTE THE ACTUAL VALUES OF THESE UNIT VECTORS IN BRF         REF         43
C                                                                B     REF         44
        CALL MAT(D3,D2,D1,1,3,2,GAMD)                                  REF         45
        CALL MATMUL(GAMD,REFMAT,DUM,3,3,3)                             REF         46
        CALL MATRAN(DUM,3,3,DUMT)                                      REF         47
        CALL MATMUL(DUMT,REFMAT,DICKUP,3,3,3)                    B     REF         48
        DO 10 I=1,7,3                                            C     REF         49
          CALL MATMUL(DICKUP,SAVE(I+770),SAVE(I+761),3,3,1)            NOSHIT      40
   10   CALL MATMUL(DICKUP,SAVE(I+27),SAVE(I+228),3,3,1)         C     REF         50
        RETURN                                                         REF         51
        END                                                            REF         52
```

P20

ROUTINE P20

SUBEXECUTIVE FOR NAVIAGATION MARKS. ADVANCES COVARIANCE, TAKES MARK, RETURNS UPDATED STATE.

AREA A — THIS AREA IS LEFT OVER FROM THE ORIGINAL VERSION AND WAS NEVER ACTIVE.

AREA B — READS UPDATED VECTOR BACK INTO ARGUMANT LOCATIONS.

```
      SUBROUTINE P20(T,Y,N)                                        P20
C                                                                  P20
      COMMON VAR                                                   P20
      DIMENSION VAR(5600), P(5000), SAVE(950), BLK(700)            P20
      EQUIVALENCE (VAR(601),P(1))                                  P20
      EQUIVALENCE (P(350),SAVE(1))                                 P20
     *,             (P(1300),BLK(1))                               P20
      DIMENSION REFMAT(3,3), X(18)                                 P20
      EQUIVALENCE (SAVE(19),REFMAT(1,1))                           P20
     *,             (SAVE(258),X(1))                               P20
C                                                                  P20
      DIMENSION Y(18)                                              P20
C     SUBROUTINE TO CONTROL THE TAKING OF A NAVIGATION MARK        P20
C                                                                  P20
C     ADVANCE W TO CURRENT TIME                                    P20
      CALL ADVW(T,Y)                                               P20
C     COMPUTE CURRENT REFMAT                             A         P20
C     CALL REF(REFMAT)                                   A         P20
C     CALCULATE AND INCORPORATE UPDATE                             P20
      CALL BVEC(N)                                                 P20
      DO 30 I=1,18                                       B         P20
   30 Y(I)=X(I)                                          B         P20
      RETURN                                                       P20
      END                                                          P20
```

ADVW

ROUTINE ADVW

ADVANCES SQUARE-ROOT MATRIX OF THE COVARIANCE. SEE REFERENCE 1 FOR DISCUSSION OF THE METHOD.

AREA A — DEFINE FINAL TIME OF W AS CURRENT TIME.
SAVE INPUT VALUE OF NUMBER OF INTEGRATED VARIABLES.
SAVE INPUT VALUE OF INTEGRATION STEP SIZE.
SET INTEGRATED VARIABLES = 13 FOR RK. SINCE BOTTOM SIX ROWS OF W ARE ASSOCIATED WITH ESTIMATED CONSTANTS, THEY NEED NOT BE INTEGRATED. RK WILL INTEGRATE TIME AND FIRST TWELVE ROWS OF W, ASSOCIATED WITH S/C AND TGT STATES.

AREA B — DEFINE TGO AS CURRENT TIME (TWF) MINUS TIME TAG ON W.
IF TGO LESS THAN 1 SECOND, GO TO AREA H. DO NOT INTEGRATE.

AREA C — TAKE EACH COLUMN OF W AND UPDATE IT:
- DEFINE TGO AS BEFORE.
- ADD COLUMN TO SAVED VALUE OF ESTIMATED STATE.
- CONVERT TO BETELGEUSE VECTOR FOR INTEGRATION.
- ENTER RKW FOR ADVANCEMENT OF ONE STEP.

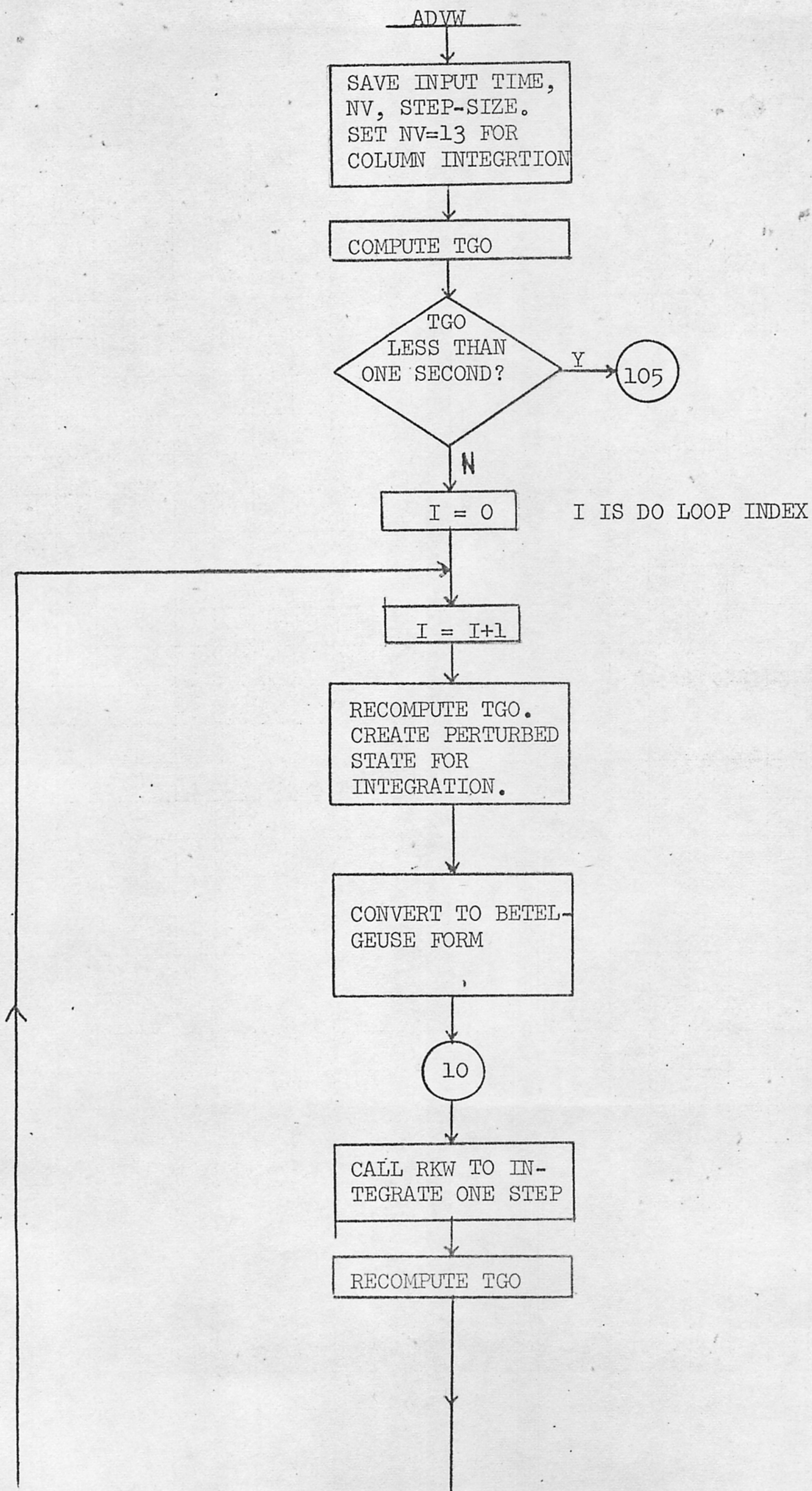
AREA D
- DECREMENT TGO BY ONE STEP.
- IF TGO LARGER THAN ONE STEP, CALL RKW.
- IF TGO LESS THAN ONE SECOND, DO NEXT COLUMN.

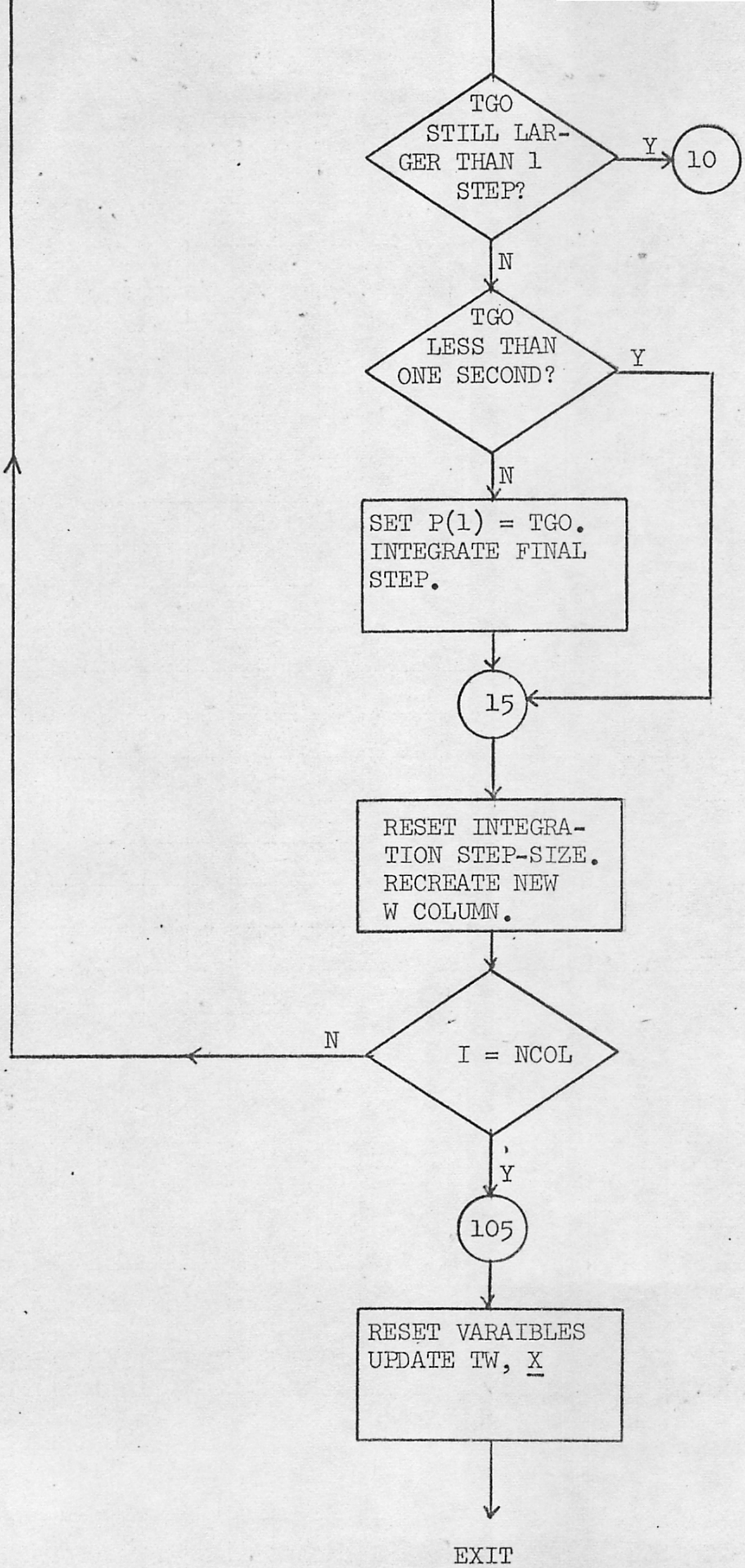
AREA E
- SET STEP EQUAL TGO FOR FINAL PASS THROUGH RKW ON THIS COLUMN.
- CALL RKW TO BRING COLUMN TO TWF.

AREA F
- RESET STEPSIZE FOR NEXT COLUMN.

AREA G
- RECONSTRUCT CARTESIAN VECTOR.
- SUBTRACT OFF CURRENT ESTMATE TO FORM NEW COLUMN OF W.
- GO ON TO NEXT COLUMN.

AREA H — ALL COLUMNS OF W ARE NOW UPDATED.
TO BE ON THE SAFE SIDE, REDEFINE PROGRAM TIME.
UPDATE TIME TAG ON W.
UPDATE SAVED STATE FOR NEXT PASS THROUGH ADVW. THIS WILL BE UPDATED AGAIN BY THE NAVIGATION MEASUREMENTS.


ADVW
SAVE INPUT TIME, NV, STEP-SIZE. SET NV=13 FOR COLUMN INTEGRTION
COMPUTE TGO
TGO LESS THAN ONE SECOND?
Y
105
N
I = 0
I IS DO LOOP INDEX
I = I+1
RECOMPUTE TGO. CREATE PERTURBED STATE FOR INTEGRATION.
CONVERT TO BETEL-GEUSE FORM
10
CALL RKW TO IN-TEGRATE ONE STEP
RECOMPUTE TGO


TGO STILL LARGER THAN 1 STEP?
Y
10
N
TGO LESS THAN ONE SECOND?
Y
N
SET P(1) = TGO. INTEGRATE FINAL STEP.
15
RESET INTEGRATION STEP-SIZE. RECREATE NEW W COLUMN.
N
I = NCOL
Y
105
RESET VARAIBLES UPDATE TW, X
EXIT

```
      SUBROUTINE ADVW(T,XF)
C
      COMMON VAR
      DIMENSION VAR(5600), Y(100), NTEGER(100), P(5000), SAVE(950),
     *          BLK(700)
      EQUIVALENCE (VAR(1),Y(1))
     *,            (VAR(401),NTEGER(1))
     *,            (VAR(601),P(1))
      EQUIVALENCE (P(350),SAVE(1))
     *,            (P(1300),BLK(1))
C
      DIMENSION UR1(3),UR2(3),UR(3)
      DIMENSION D(10), X(18), WE(18,27)
      EQUIVALENCE (SAVE(9),D(1))
     *,            (SAVE(258),X(1))
     *,            (SAVE(276),WE(1,1))
C
      EQUIVALENCE (D(1),TW)
C
      EQUIVALENCE (D(2),UR1(1)),(D(5),UR2(1))
     *,            (BLK(649),UR(1))
C
      DIMENSION XOP(12),XF(18),XFP(12)
      EQUIVALENCE (BLK(1),XOP(1)),(BLK(13),XFP(1))
      EQUIVALENCE (NTEGER(9),NCOL)
C
      COL = NCOL
C     SAVE CURRENT TIME NO. INT. VAR. STEP SIZE, SET I.V.=13
      TWF=T                                                     A
      NV=NTEGER(6)
      STEP=P(1)
      NTEGER(6)=13                                              A
C     CHECK TGO                                                 B
      TGO=TWF-TW
      IF(ABS(TGO).LT.1.) GO TO 105                              B
C     ADVANCE W BY COLUMNS                                      C
      DO 100 I=1,NCOL
      TGO=TWF - TW
C     CREATE PERTURBED STATE
      DO 5 J=1,12
    5 XOP(J) = X(J) + WE(J,I)*SQRT(COL)
C     CONVERT TO BETELGEUSE SYSTEM
      CALL CART2(XOP,Y(2))
```

```
   10   CALL RKW                                          C        ADVW
        TGO=TGO - P(1)                                    D        ADVW
        IF(TGO.GE.P(1)) GO TO 10                                   ADVW
        IF(ABS(TGO).LT.1.) GO TO 15                       D        ADVW
        P(1)=TGO                                          E        ADVW
        CALL RKW                                          E        ADVW
   15   P(1)=STEP                                         F        ADVW
C       RECTIFY RESULTING STATE AND COMPUTE NEW COLUMN OF W  G     ADVW
        CALL CART1(XFP,Y(2))                                       ADVW
        DO 20 J=1,12               WORKS                           ADVW
   20   WE(J,I) = (XFP(J) - XF(J))/SQRT(COL)              G        ADVW
  100   CONTINUE                                                   ADVW
```

```
UBROUTINE  ADVW                      CDC 6600 FTN V3.0-P308 OPT=1  08/29/72  11.32.27.

  105   CONTINUE                                                   ADVW
        Y(1)=TWF                                          H        ADVW
        TW=TWF                                                     ADVW
        NTEGER(6)=NV                                               ADVW
        DO 110 I=1,18                                              ADVW
  110   X(I)=XF(I)                                        H        ADVW
        RETURN                                                     ADVW
        END                                                        ADVW
```

DVEC

ROUTINE BVEC

COMPUTES GEOMETRY VECTORS AS SPECIFIED BY CALLING ROUTINE. CALLS FILTER TO UPDATE STATE AND COVARIANCE. SEE REFERENCE 1 FOR DISCUSSION.

AREA RR COMPUTES RANGE RATE MEASUREMENT GEOMETRY VECTOR:

$\underline{r}$ = [DX(1), DX(2), DX(3)] RELATIVE POSITION VECTOR

$\underline{v}$ = [DX(4), DX(5), DX(6)] RELATIVE VELOCITY VECTOR

$\underline{UR} = UNIT(\underline{r})$

RC = RANGE

$$\underline{B}_{RD} = \begin{bmatrix} -\underline{UR} \times (\underline{v} \times \underline{UR})/RC \\ -\underline{UR} \\ \underline{UR} \times (\underline{v} \times \underline{UR})/RC \\ \underline{UR} \end{bmatrix}$$

AREA A CALCULATE THE ESTIMATED RELATIVE POSITION VECTOR.

AREA B DEFINE CURRENT ESTIMATED RANGE-RATE AS $\underline{UR}^T\underline{v}$ PLUS RANGE-RATE BIAS ESTIMATE.

AREA C DEFINE CURRENT ESTIMATED RANGE AS $\underline{UR}^T\underline{r}$ PLUS RANGE BIAS ESTIMATE.

AREA D COMPUTE $\underline{UR} \times (\underline{v} \times \underline{UR})$ USING PORTIONS OF GEOMETRY VECTOR AS SCRATCH PAD.

AREA E LOAD B AS DEFINED ABOVE (AREA RR). B(14) IS THE PARTIAL OF R-RATE MEASUREMENT WITH RESPECT TO A BIAS ESTIMATE ERROR: SET EQUAL TO 1.

AREA F DEFINE MEASUREMENT RESIDUAL (DQ) AS MEASURED R-RATE MINUS ESTIMATED.
DEFINE R-RATE SIGMA AS MAX OF R-RATE FRACTION AND MINIMUM VALUE.

AREA F   SET MEASUREMENT TYPE FLAG FOR FILTER
UPDATE STATE WITH R-RATE MEASUREMENT (CALL FILTER)

AREA R   COMPUTES RANGE MEASUREMENT GEOMETRY VECTOR:

$$\underline{B}_R = \begin{bmatrix} -\underline{UR} \\ \underline{0}_3 \\ \underline{UR} \\ \underline{0}_3 \end{bmatrix}$$

AREA A   COMPUTE CURRENT RANGE UNIT VECTOR.

AREA B   DEFINE ESTIMATED RANGE AS IN AREA RR(C)

AREA C   LOAD B AS DEFINED ABOVE. B(13) IS THE PARTIAL OF A RANGE MEASUREMENT WITH RESPECT TO A RANGE BIAS ESTIMATE ERROR: SET EQUAL TO 1.

AREA D   DEFINE MEASUREMENT RESIDUAL AS MEASURED RANGE MINUS ESTIMATED RANGE.
DEFINE RANGE SIGMA AS MAX OF RANGE FRACTION AND MINIMUM VALUE.
SET MEASUREMENT TYPE FLAG FOR FILTER.
UPDATE STATE WITH RANGE MEASUREMENT (CALL FILTER)

AREA E   IF THIS IS A RANGE-ONLY (VHF) MARK, EXIT ROUTINE.

AREA OP   COMPUTES AZIMUTH AND ELEVATION MEASUREMENT GEOMETRY VECTORS:

$\underline{REF1}$, $\underline{REF2}$, $\underline{REF3}$ ESTIMATED NAV BASE UNIT VECTORS.

$$AZ = ATAN[(\underline{UR}^T \underline{REF3})/(\underline{UR}^T \underline{REF2})]$$

$$EL = ASIN[\underline{UR}^T \underline{REF1}]$$

THESE DEFINITIONS ARE NOT UNIQUE, BUT THEY MUST BE THE SAME AS THOSE IN SUBROUTINE REL.

AREA OP FOR AZ: $RCEL = \underline{UR}^T \underline{r}[COS(EL)]$ PROJECTION OF RANGE INTO REF2, REF3 PLANE.

$$\underline{B}_{AZ} \begin{bmatrix} -UNIT(\underline{REF1} \times \underline{UR})/RCEL \\ \underline{0}_3 \\ UNIT(REF1 \times UR)/RCEL \\ \underline{0}_3 \end{bmatrix}$$

FOR EL: $RCEL = \underline{UR}^T \underline{r}$ RANGE MAGNITUDE

$$\underline{B}_{EL} \begin{bmatrix} -UNIT[\underline{UR} \times (\underline{REF1} \times \underline{UR})]/RCEL \\ \underline{0}_3 \\ UNIT[\underline{UR} \times (\underline{REF1} \times \underline{UR})]/RCEL \\ \underline{0}_3 \end{bmatrix}$$

AREA A COMPUTE THE CURRENT ESTIMATES OF AZ AND EL AS DEFINED ABOVE, INCLUDING CURRENT ESTIMATES OF BIASES X(15), X(16).

AREA B IF ELEVATION COMPONENT OF OPTICS MARK, GO TO AREA F TO COMPUTE GEOMETRY VECTOR.

AREA C COMPUTE AZIMUTH GEOMETRY VECTOR AS ABOVE.

AREA D DEFINE MEASUREMENT RESIDUAL.
DEFINE AZ SIGMA

AREA E DEFINE RANGE PROJECTION TERM.
GO TO AREA I TO LOAD B AND TAKE MARK.

AREA F DEFINE ELEVATION GEOMETRY VECTOR AS DESCRIBED ABOVE.

AREA G DEFINE ELEVATION MEASUREMENT RESIDUAL.
DEFINE ELEVATION SIGMA.

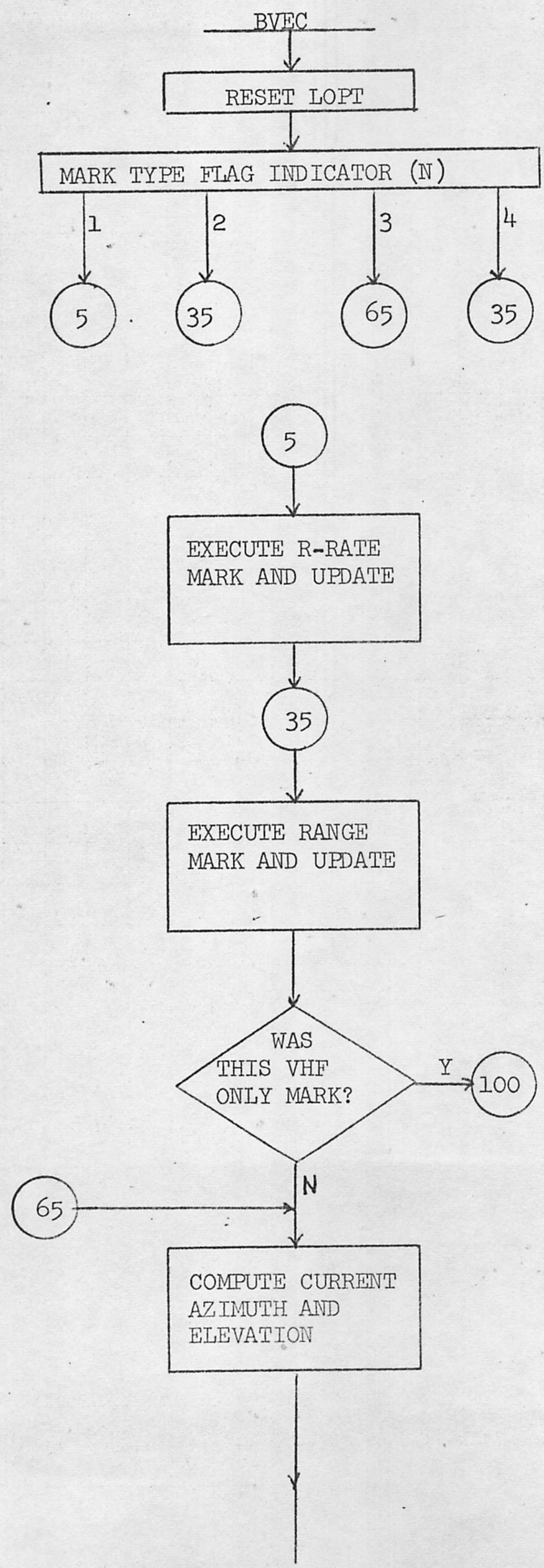
AREA H DEFINE RANGE PROJECTION TERM.

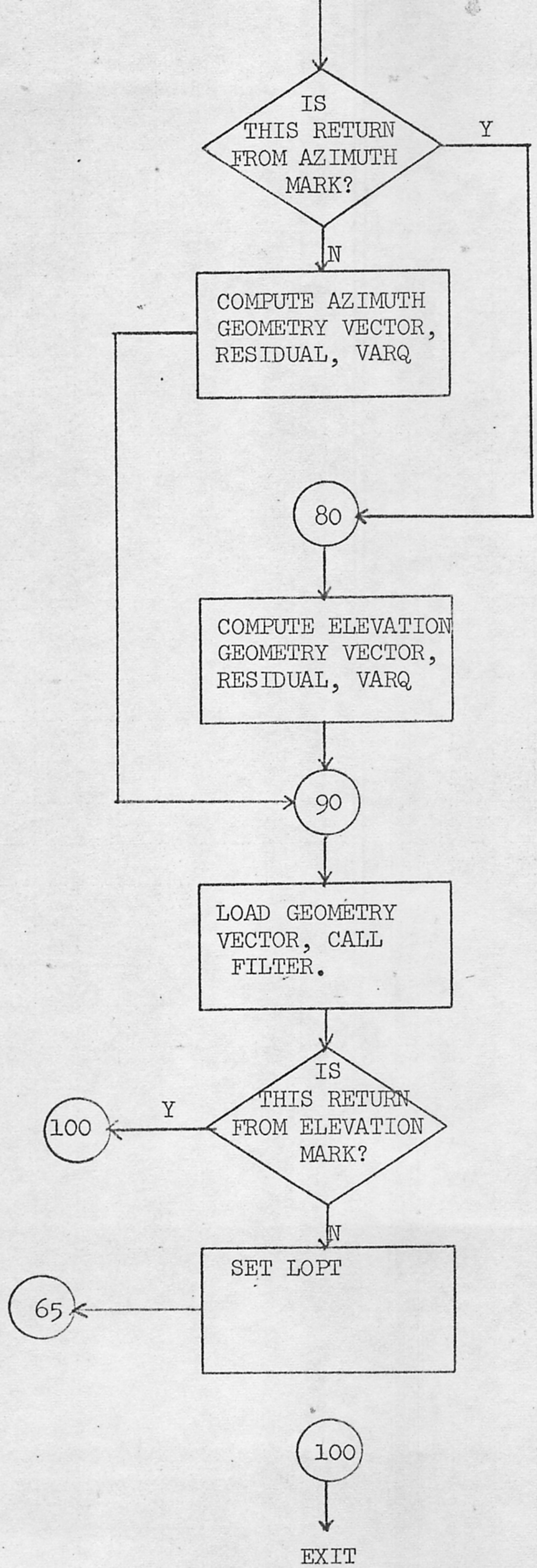
AREA I LOAD B WITH GEOMETRY VECTOR AS DESCRIBED ABOVE.

AREA J B(16) IS PARTIAL OF ELEVATION ANGLE MEASUREMENT WITH RESPECT TO BIAS ESTIMATE ERROR. IF THIS IS ELEVATION COMPONENT, SET B(16)=1.
B(15) IS PARTIAL OF AZIMUTH ANGLE MEASUREMENT WITH RESPECT TO BIAS ESTIMATE ERROR. IF THIS IS AZIMUTH COMPONENT, SET B(15)=1.

AREA K SET APPROPRIATE MEASUREMENT TYPE FLAG FOR FILTER.
CALL FILTER TO TAKE MARK.

AREA L IF ELEVATION MARK HAS JUST BEEN PROCESSED, EXIT ROUTINE.
LOPT FLAG WILL BE RESET ON NEXT ENTRY TO BVEC.
IF THIS WAS NOT AN ELEVATION MARK, GO TO AREA OP TO PERFORM THIS MEASUREMENT.


BVEC
RESET LOPT
MARK TYPE FLAG INDICATOR (N)
1
2
3
4
5
35
65
35
5
EXECUTE R-RATE MARK AND UPDATE
35
EXECUTE RANGE MARK AND UPDATE
WAS THIS VHF ONLY MARK?
Y
100
N
65
COMPUTE CURRENT AZIMUTH AND ELEVATION


IS THIS RETURN FROM AZIMUTH MARK?
Y
N
COMPUTE AZIMUTH GEOMETRY VECTOR, RESIDUAL, VARQ
80
COMPUTE ELEVATION GEOMETRY VECTOR, RESIDUAL, VARQ
90
LOAD GEOMETRY VECTOR, CALL FILTER.
IS THIS RETURN FROM ELEVATION MARK?
Y
100
N
SET LOPT
65
100
EXIT

```
      SUBROUTINE BVEC(N)
C
C     SUBROUTINE TO COMPUTE GEOMETRY VECTORS FOR NAVIGATION MARKS
C
      COMMON VAR
      DIMENSION VAR(5600), NTEGER(100), P(5000), SAVE(950), BLK(700)
      EQUIVALENCE (VAR(401),NTEGER(1))
     *,                 (VAR(601),P(1))
      EQUIVALENCE (NTEGER(29),NGUIDE)
      EQUIVALENCE (P(350),SAVE(1))
     *,                 (P(1300),BLK(1))
      DIMENSION QQ(4), SIG(4), C(10), REFMAT(3,3), XNBN(3), YNBN(3)
     *,          ZNBN(3), NE(10), NM(10), DTL(10), DTN(10), DTM(10)
     *,          USP(10), USV(10), UTP(10), UTV(10), SR(10), SRD(10)
     *,          SO(10), SC1(10), SC2(10), NW(10), TLM(10), NS(3)
     *,          ZTZ(4), SZ(4), TALIGN(10), XNBE(3), YNBE(3), ZNBE(3)
     *,          X(18), WE(18,27)
      EQUIVALENCE (SAVE(1),QQ(1)),            (SAVE(5),SIG(1))
     *,            (SAVE(9),C(1)),             (SAVE(19),REFMAT(1,1))
     *,            (SAVE(28),XNBN(1)),         (SAVE(31),YNBN(1))
     *,            (SAVE(34),ZNBN(1)),         (SAVE(37),NE(1))
     *,            (SAVE(47),NM(1)),           (SAVE(57),DTL(1))
     *,            (SAVE(67),DTN(1)),          (SAVE(77),DTM(1))
     *,            (SAVE(87),USP(1)),          (SAVE(97),USV(1))
     *,            (SAVE(107),UTP(1)),         (SAVE(117),UTV(1))
     *,            (SAVE(127),SR(1)),          (SAVE(137),SRD(1))
     *,            (SAVE(147),SO(1)),          (SAVE(157),SC1(1))
     *,            (SAVE(167),SC2(1)),         (SAVE(177),NW(1))
     *,            (SAVE(187),TLM(1)),         (SAVE(197),NS(1))
     *,            (SAVE(200),ZTZ(1)),         (SAVE(204),SZ(1))
     *,            (SAVE(208),TALIGN(1)),      (SAVE(218),NALIGN)
     *,            (SAVE(229),XNBE(1)),        (SAVE(232),YNBE(1))
     *,            (SAVE(235),ZNBE(1)),        (SAVE(258),X(1))
     *,            (SAVE(276),WE(1,1))
      DIMENSION REF1(3), REF2(3), REF3(3)
      EQUIVALENCE (XNBN(1),REF1(1))
     *,            (YNBN(1),REF2(1))
     *,            (ZNBN(1),REF3(1))
C
      EQUIVALENCE (C(1),TW)
C
      DIMENSION B(18), DX(6), UR(3), WV(18), BZ(27), WS(27,18)
```

```
      EQUIVALENCE (BLK(1),B(1))
     *,           (BLK(19),DX(1))
     *,           (BLK(25),UR(1))
     *,           (BLK(28),VARQ)
     *,           (BLK(29),DQ)
     *,           (BLK(30),NINS)
     *,           (BLK(31),WV(1))
     *,           (BLK(49),BZ(1))
     *,           (BLK(76),WS(1))
C
      COMMON/BV/RVAR,RVARMIN,VVAR,VVARMIN,VARAZ,VAREL
C
```



```
      LOPT=0
C     CHECK IF RADAR (N=1), VHF (N=2), OR OPTICS (N=3) MARK IS TO DONE
      GO TO(5,35,65,35) N
C
    5 CONTINUE                                          RR
C     COMPUTE RELATIVE STATE
      DO 10 I=1,6
   10 DX(I)=X(I+6)-X(I)
C     COMPUTE UNIT RANGE VECTOR                          A
      CALL UVEC(DX(1),DX(2),DX(3),UR)
C     COMPUTE CURRENT ESTIMATED RANGE RATE               A
      RDC=DOT(UR,DX(4)) + X(14)                          B
C     COMPUTE CURRENT ESTIMATED RANGE                    C
      RC=DOT(UR,DX(1)) + X(13)                           C
C     COMPUTE RANGE RATE GEOMETRY VECTOR                 D
      CALL CROSS(DX(4),UR,B(4))
      CALL CROSS(UR,B(4),B(1))                           D
C     LOAD B                                             E
      DO 15 I=1,3
      B(I)=-B(I)/RC
      B(I+3)=-UR(I)
      B(I+6)=-B(I)
      B(I+9)=UR(I)
      B(I+12)=0.
```

```
      B(14)=1.
C     DEFINE VARQ AND DQ
      DQ = QQ(2) - RDC
      VARQ=AMAX1(ABS(RDC*VVAR),VVARMIN)**2
      NINS=2
C     CALCULATE AND INCORPORATE UPDATE
      CALL FILTER
   35 CONTINUE
C     COMPUTE RELATIVE STATE FOR VHF MEASUREMENT
      DO 40 I=1,6
   40 DX(I)=X(I+6)-X(I)
C     COMPUTE CURRENT UNIT RANGE VECTOR
      CALL UVEC(DX(1),DX(2),DX(3),UR)
C     COMPUTE CURRENT RANGE ESTIMATE
      RC=DOT(UR,DX(1)) + X(13)
C     LOAD B
      DO 45 I=1,3
      B(I)=-UR(I)
      B(I+3)=0.
      B(I+6)=UR(I)
      B(I+9)=0.
      B(I+12)=0.
   45 B(I+15)=0.
      B(13)=1.
C     DEFINE VARQ, DQ
      DQ = QQ(1) - RC
      VARQ=AMAX1(RC*RVAR,RVARMIN)**2
      NINS=1
C     CALCULATE AND INCORPORATE MARK
      CALL FILTER
```



```
C     CHECK IF THIS WAS A VHF MARK OR SECOND PART OF RADAR MARK
      IF(N.EQ.2) GO TO 100
   65 CONTINUE
C     COMPUTE RELATIVE STATE FOR OPTICS MARK
      DO 70 I=1,6
   70 DX(I)=X(I+6) - X(I)
```

```
      CALL UVEC(DX(1),DX(2),DX(3),UR)
C     COMPUTE CURRENT AZIMUTH ESTIMATE                               A
      AZC=ATAN(DOT(UR,REF3)/DOT(UR,REF2)) + X(15)
      COMPUTE CURRENT ESTIMATED ELEVATION
      ELC=ASIN(DOT(UR,REF1)) + X(16)                                 A
C     CHECK IF ELEVATION COMPONENT IS CURRENTLY BEING PROCESSED
      IF(LOPT.EQ.1) GO TO 80                                         B
C     COMPUTE AZIMUTH GEOMETRY VECTOR                                C
      CALL UCROSS(REF1,UR,B(1))                                      C
C     DEFINE VARQ, DQ AND PROJECTION TERM                            D
      DQ = DQ(3) - AZC
      VARQ=VARAZ**2                                                  D
      RCEL=DOT(UR,DX(1))*COS(ELC)                                    E
C     LOAD B AND INCORPORATE MARK
      GO TO 90
   80 CONTINUE                                                       F
C     DEFINE ELEVATION GEOMETRY VECTOR
      CALL UCROSS(REF1,UR,B(4))
      CALL UCROSS(UR,B(4),B(1))                                      F
C     DEFINE VARQ, DQ AND PROJECTION TERM                            G
      DQ = DQ(4) - ELC
      VARQ=VAREL**2                                                  G
      RCEL=DOT(UR,DX(1))                                             H
   90 CONTINUE                                                       I
C     LOAD B
      DO 95 I=1,3
      B(I)=-B(I)/RCEL
      B(I+3)=0
      B(I+6)=-B(I)
      B(I+9)=0.
      B(I+12)=0.
   95 B(I+15)=0.                                                     I
      IF(LOPT.EQ.1) B(16)=1.                                         J
      IF(LOPT.EQ.0) B(15)=1.                                         J
      IF(LOPT.EQ.1) NINS=4                                           K
      IF(LOPT.EQ.0) NINS=3                                           K
      CALL FILTER
C     CHECK IF THIS WAS THE AZIMUTH COMPONENT OF THE OPTICS MARK
      IF(LOPT.EQ.1) GO TO 100                                        L
      LOPT=1
      GO TO 65
  100 CONTINUE                                                       L
      RETURN
      END
```

FILTER

ROUTINE FILTER

COMPUTES OPTIMAL WEIGHTING VECTOR FOR MEASUREMENT, UPDATES STATE AND COVARIANCE. FOR DISCUSSION, SEE REFERENCE 1.

$$WE = \text{COVARIANCE SQUARE-ROOT MATRIX}$$

$$WS = WE^T$$

$$\underline{BZ} = WS \times \underline{B}$$

$A = VARQ + \underline{BZ}^T \underline{BZ}$	TOTAL A PRIORI UNCERTAINTY IN MEASUREMENT.
$ZTZ = \underline{BZ}^T \underline{BZ}$	TOTAL A PRIORI UNCERTAINTY IN MEASUREMENT DUE TO STATE UNCERTAINTY.
$SZ = [VARQ/A]^{\frac{1}{2}}$	CONFIDENCE LEVEL OF A PRIORI ESTIMATE OF MEASUREMENT.
$\underline{WV} = WE \times \underline{BZ}$	WEIGHTING VECTOR.
$\underline{X} = \underline{X} + (DQ/A)\underline{WV}$	STATE UPDATE EQUATION.
$WE = WE - (1/VARB)\underline{WV}\ \underline{BZ}^T$	COVARIANCE UPDATE EQUATION.

$$VARB = A(1. + SZ)$$

AREA A CALCULATE TRANSPOSE OF W.
CALCULATE Z-VECTOR.

AREA B CALCULATE A.

AREA C CALCULATE ZTZ FOR THIS MARK COMPONENT (FOR POPOUT)

AREA D CONVERT ZTZ TO MR IF ANGLE MARK.
CALCULATE SZ FOR POPOUT.

AREA E CALCULATE WEIGHTING VECTOR.

AREA F CALCULATE COVARIANCE UPDATE FACTOR.

AREA G UPDATE STATE AND COVARIANCE.

```
      SUBROUTINE FILTER
C
C     SUBROUTINE TO PROCESS A NAVIGATION MARK
C
      COMMON VAR
      DIMENSION VAR(5600), P(5000), SAVE(950), BLK(700)
     *,          NTEGER(100)
      EQUIVALENCE (VAR(601),P(1))
     *,          (VAR(401),NTEGER(1))
      EQUIVALENCE (P(350),SAVE(1))
     *,          (P(1300),BLK(1))
      DIMENSION ZTZ(4), SZ(4), X(18), WE(18,27)
      EQUIVALENCE (SAVE(200),ZTZ(1))
     *,          (SAVE(204),SZ(1))
     *,          (SAVE(258),X(1))
     *,          (SAVE(276),WE(1,1))
     *,          (NTEGER(9),NCOL)
C
C
      DIMENSION B(18), DX(6), UR(3), WV(18), BZ(27), WS(27,18)
     *,          EBAR(18)
C
      EQUIVALENCE (BLK(1),B(1))
     *,          (BLK(19),DX(1))
     *,          (BLK(25),UR(1))
     *,          (BLK(28),VARQ)
     *,          (BLK(29),DQ)
     *,          (BLK(30),NINS)
     *,          (BLK(31),WV(1))
     *,          (BLK(49),BZ(1))
     *,          (BLK(76),WS(1))
     *,          (BLK(562),EBAR(1))
C
C     COMPUTE Z-VECTOR
      CALL MATRAN(WE,18,27,WS)                                    A
      CALL MATMUL(WS,B,BZ,27,18,1)                                A
      A=VARQ                                                      B
      DO 5 I=1,27
    5 A=A + BZ(I)*BZ(I)                                           B
C     COMPUTE STATE UNCERTAINTY OF OBSERVABLE AND REDUCTION FACTOR
      ZTZ(NINS)=SQRT(A-VARQ)                                      C
      IF(NINS.GT.2) ZTZ(NINS)=ZTZ(NINS)*1000.                     D
      SZ(NINS)=SQRT(VARQ/A)                                       D
```

```
      CALL MATMUL(WE,BZ,WV,18,27,1)                        E    FILTER      45
      VARB=A*(1. + SQRT(VARQ/A))                           F    FILTER      46
      DO 10 I=1,18                                         G    FILTER      47
      X(I)=X(I) + WV(I)*DQ/A                                    FILTER      48
      DO 10 J=1,27                                              FILTER      49
   10 WE(I,J)=WE(I,J) - WV(I)*BZ(J)/VARB                   G    FILTER      50
      RETURN                                                    FILTER      51
      END                                                       FILTER      52
```

OREN1 -OPEN5

6.0 REFERENCES

1. EVERYTHING YOU ALWAYS WANTED TO KNOW... (ENCLOSURE)

2. BATTIN, R. H., ASTRONAUTICAL GUIDANCE, NEW YORK, 1964

EVERYTHING YOU ALWAYS WANTED TO KNOW ABOUT KALMAN FILTERING
(AREN'T YOU GLAD YOU ASKED)

I. Fundamental definitions and important properties of vectors and matrices

A matrix A is said to be an nxm array when it has n rows and m columns.

An nx1 matrix is called a (column) vector

A 1xn matrix is called a (row) vector

If n = m, A is square.

a_{ij} is a representative element of A from the i^{th} row and j^{th} column

Define $\underline{b}$ an arbritrary nx1 (vector)

A an arbritrary nxn (square)

B an arbritrary nxm (rectangular)

Further

$$\underline{0} = \begin{bmatrix} 0 \\ 0 \\ - \\ - \\ - \\ 0 \end{bmatrix} \quad (n\times 1) \qquad I = \begin{bmatrix} 1 & 0 & 0 & - & - & 0 \\ 0 & 1 & 0 & - & - & 0 \\ 0 & 0 & 1 & - & - & 0 \\ - & & & - & & - \\ - & & & & - & - \\ 0 & 0 & 0 & - & - & 1 \end{bmatrix} \quad (n\times n)$$

$$A\underline{0} = \underline{0} \qquad AI = A$$

For $A = \left[a_{ij}\right]$, $A^T = \left[a_{ji}\right]$ transpose of A

A^{-1} is the matrix, if it exists, which has the property that

$A^{-1}A = AA^{-1} = I$, inverse of A

Properties of Matrices

If A is a square matrix, and if for all $\underline{b} \neq \underline{0}$,

$\underline{b}^T A \underline{b} > 0$ A is positive definite (p.d.)

$\underline{b}^T A \underline{b} < 0$ A is negative definite (n.d.)

If, for any $\underline{b} \neq \underline{0}$,

$\underline{b}^T A \underline{b} = 0$ A is indefinite

If $\qquad A = A^T \qquad$ A is <u>symmetric</u>

$$A^T = A^{-1} \qquad \text{A is orthogonal}$$

If A, B are square

$$(AB)^T = B^T A^T$$

$$(AB)^{-1} = B^{-1} A^{-1}$$

<u>Properties of Vectors</u>

If for a set $\{\underline{b}_i\}$ of nx1 vectors

$$\sum_i c_i \underline{b}_i = \underline{0} \Rightarrow c_i = 0$$

The $\underline{b}_i$ are said to be <u>linearly independent</u>. If there are n members of the set $\{\underline{b}_i\}$, the set is <u>complete</u> with respect to the space of nx1 vectors. If the set $\{\underline{b}_i\}$ is independent and complete, any arbitrary nx1 vector, $\underline{e}$, may be expressed as

$$\underline{e} = \sum_i c_i \underline{b}_i$$

Given two vectors, $\underline{b} \neq \underline{0}$, $\underline{c} \neq \underline{0}$, if

$$\underline{b}^T \underline{c} = 0 \qquad \underline{b} \text{ and } \underline{c} \text{ are orthogonal}$$

If

$$\underline{b}^T \underline{b} = 1 \qquad \underline{b} \text{ is normal}$$

<u>Eigenvalues, eigenvectors, functions of a matrix</u>

If there exists a set of vectors $\underline{L}_i$ and corresponding numbers λ_i such that

$$A\underline{L}_i = \lambda_i \underline{L}_i$$

$\underline{L}_i$ is said to be an eigenvector of A corresponding to eigenvalve λ_i.

If A is definite ($\underline{b}'A\underline{b} \neq 0$ for all $\underline{b} \neq \underline{0}$) it has no $\lambda_i = 0$:

Suppose for some $\underline{b}_i$, $\lambda_i = 0$

$$\underline{b}_i{}^T A \underline{b}_i = \lambda_i \underline{b}_i{}^T \underline{b}_i$$
$$= 0$$

contrary to the assumption.

In addition, if A is definite, its eigenvectors are a complete linearly independent set. In this case, A may be represented as

$$A = P \Lambda P^{-1}$$

$$P = \begin{bmatrix} \underline{L}_1 & \underline{L}_2 & - & - & \underline{L}_n \end{bmatrix} \qquad \Lambda = \begin{bmatrix} \lambda_1 & 0 & - & - & 0 \\ 0 & \lambda_2 & - & - & 0 \\ - & & - & & 1 \\ - & & & - & - \\ 0 & 0 & - & - & \lambda_n \end{bmatrix}$$

square matrix of eigenvectors

diagonal matrix of eigenvalues

Furthermore, the function

$$f(A) = \sum_i c_i A^i$$

exists and converges to

$$f(A) = P \begin{bmatrix} \sum c_i \lambda_1{}^i & 0 & - & - & 0 \\ 0 & \sum c_i \lambda_2{}^i & - & - & 0 \\ - & - & & & - \\ - & - & - & & - \\ 0 - & 0 & - & - & \sum c_i \lambda_n{}^i \end{bmatrix} P^{-1}$$

$$= P \, f(\Lambda) \, P^{-1}$$

This may be demonstrated by noting that

$$AP = \left[\lambda_1 \underline{L}_1 \;\; \lambda_2 \underline{L}_2 \; - \; - \; \lambda_n \underline{L}_n \right] = P\Lambda$$

$$A^i P = \left[\lambda_1^i \; \underline{L}_1 \lambda_2^i \; \underline{L}_2 \; - \; - \; \lambda_n^i \; \underline{L}_n \right] = P\Lambda^i$$

$$A^i = P\Lambda^i P^{-1}$$

The normalized eigenvectors of a symmetric matrix are orthonormal. Let $\{\underline{e}_i\}$ be the normalized e. v. of E, a symmetric matrix

$$\underline{e}_j{}^T E \underline{e}_i = \lambda_i \underline{e}_j{}^T \underline{e}_i$$

$$e_j{}^T E \underline{e}_i = (E^T \underline{e}_j)^T \underline{e}_i$$

$$= (E\underline{e}_j)^T \underline{e}_i$$

$$= \lambda_j \underline{e}_j{}^T \underline{e}_i$$

$$e_j{}^T E \underline{e}_i - \underline{e}_j{}^T E \underline{e}_i = 0 = (\lambda_i - \lambda_j)\, \underline{e}_j{}^T \underline{e}_i$$

If $\lambda_i \neq \lambda_j$ it is seen that $\underline{e}_j{}^T \underline{e}_i = 0$. If $\lambda_i = \lambda_j$, one may construct a new vector, $\underline{e}_j{}^*$ which has an eigenvalue λ_i and is orthogonal to $\underline{e}_i$:

$$\underline{e}_j{}^* = (\underline{e}_j - (\underline{e}_i{}^T \underline{e}_j)\, \underline{e}_i$$

$$\underline{e}_i{}^T e_j{}^* = \underline{e}_i{}^T \underline{e}_j - (\underline{e}_i{}^T \underline{e}_j)\, \underline{e}_i{}^T \underline{e}_i$$

$$= \underline{e}_i{}^T \underline{e}_j - (\underline{e}_i{}^T \underline{e}_j) = 0$$

Since $\underline{e}_i{}^T \underline{e}_i = 1$ (normalized). This may be done for every repeated eigenvalue until a complete orthonormal set is constructed.

II. Conventional usages in vector and matrix calculus

Differentiation with respect to a scalar:

$$\frac{d}{dt}A = \dot{A} = \left[\frac{d}{dt}a_{ij}\right]$$

Differentiation with respect to a vector:

$$\underline{X} = \{X_i\} \qquad \text{nx1 vector}$$

$$\Phi = \Phi(\underline{X}) \qquad \text{a scalar}$$

For a scalar

$$\frac{\partial\Phi}{\partial\underline{X}} = \left[\frac{\partial\Phi}{\partial X_i}\right] = \begin{bmatrix} \frac{\partial\Phi}{\partial X_1} \\ \frac{\partial\Phi}{\partial X_2} \\ - \\ - \\ \frac{\partial\Phi}{\partial X_n} \end{bmatrix}$$

For a vector $\underline{R} = \{R_i\}$

$$\frac{\partial \underline{R}}{\partial \underline{X}} = \begin{bmatrix} \frac{\partial R_1}{\partial X_1} & \frac{\partial R_2}{\partial X_1} & - & - & - & \frac{\partial R_n}{\partial X_1} \\ \frac{\partial R_1}{\partial X_2} & \frac{\partial R_2}{\partial X_2} & - & - & - & \frac{\partial R_n}{\partial X_2} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ \frac{\partial R_1}{\partial X_n} & \frac{\partial R_2}{\partial X_n} & & & & \frac{\partial R_n}{\partial X_n} \end{bmatrix}^T$$

$$= \begin{bmatrix} \frac{\partial R_1}{\partial X_1} & \frac{\partial R_1}{\partial X_2} & - & - & - & \frac{\partial R_1}{\partial X_n} \\ \frac{\partial R_2}{\partial X_1} & \frac{\partial R_2}{\partial X_2} & - & - & - & \frac{\partial R_2}{\partial X_n} \\ - & - & & & & \\ - & - & & & & \\ - & - & & & & \\ \frac{\partial R_n}{\partial X_1} & \frac{\partial R_n}{\partial X_2} & - & - & - & \frac{\partial R_n}{\partial X_n} \end{bmatrix}$$

$$= \left\{\frac{\partial}{\partial \underline{X}} (\underline{R}^T)\right\}^T$$

Integration of a matrix:

$$\int A dt = \left[\int a_{ij} dt\right]$$

Differentials:

$$\delta (\ ) = \frac{\partial (\ )}{\partial \underline{X}} \delta \underline{X}$$

Examples:

$$\delta \Phi = \frac{\partial \Phi}{\partial \underline{X}} \delta \underline{X}$$

$$\delta R = \frac{\partial \underline{R}}{\partial \underline{X}} \delta \underline{X} = \left\{ \frac{\partial}{\partial \underline{X}} \underline{R}^T \right\}^T \delta \underline{X}$$

III. Eyeball statistics

Let X be a discrete random variable with sample described by the distribution $\{X_i\}$

$$\text{Average of } X = \overline{X} = \frac{1}{N} \sum_i X_i$$

$$\text{Variance of } X = \text{var}(X) = \sigma_X{}^2 = \frac{1}{N} \sum_i (X_i - \overline{X})^2 = \overline{(X_i - \overline{X})^2}$$

$$\text{Standard deviation of } X = \sigma X = \left[\text{var}(X) \right]^{1/2}$$

If X is a continuous variable, the distribution is characterized by $f(\xi)$, the probability per unit interval that $X = \xi$. In this case,

$$\overline{X} = \int_{-\infty}^{\infty} \xi f(\xi) d\xi$$

$$\text{Var}(X) = \int_{-\infty}^{\infty} (\xi - \overline{X})^2 f(\xi)\, d\xi$$

For the vector $\underline{X}$, a joint distribution function is defined $f(\underline{\xi})$

$$\overline{\underline{X}} = \int_V \underline{\xi}\, f(\underline{\xi})\, d\xi_1 d\xi_2 - - d\xi_n$$

$$\mathrm{COV}(\underline{X}) = \overline{\left[(\underline{X}-\overline{\underline{X}})(\underline{X}-\overline{\underline{X}})^T\right]} \text{ covariance matrix of } X$$

$$= \int_V (\underline{\xi}-\overline{\underline{X}})(\underline{\xi}-\overline{\underline{X}})^T f(\underline{\xi})\, d\xi_1\, d\xi_2 - - d\xi_n$$

$$= \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} & - & - & - & \sigma_{x_1 x_n} \\ \sigma_{x_2 x_1} & \sigma_{x_2}^2 & - & - & - & \sigma_{x_2 x_n} \\ - & - & & & & \\ - & - & & & & \\ - & - & & & & \\ \sigma_{x_n x_1} & \sigma_{x_n x_2} & - & - & - & \sigma_{x_n}^2 \end{bmatrix}$$

where $\sigma_{x_i x_j}$ = covariance of x_i, x_j

$$= \int_V (\xi_i - \bar{x}_i)(\xi_j - \bar{x}_j)\, f(\underline{\xi})\, d\xi_1 d\xi_2 - - d\xi_n$$

If $\sigma_{x_i x_j} = 0$, x_i and x_j are said to be linearly independent in the statistical sense. It <u>does not imply general functional independence</u>.

The function

$$\rho_{ij} = \frac{\sigma_{x_i x_2}}{\sqrt{\sigma_{x_i}^2}\ \sqrt{\sigma_{x_j}^2}}$$

is called the correlation coefficient of x_i to x_j.

For physical systems, covariance matrices are virtually always positive definite.

Let $\underline{e} = \underline{X} - \overline{\underline{X}}$ and

$$E = \overline{\underline{e}\underline{e}^T} = \text{COV}(X)$$

E is symmetric:

$$E^T = \overline{\underline{e}\underline{e}^T}^{\,T} = \overline{(\underline{e}\underline{e}^T)^T} = \overline{\underline{e}\underline{e}^T} = E$$

The matrix of normalized eigenvectors of E is orthogonal, i.e. $P^T = P^{-1}$:

Since E is symmetric, its eigenvectors are orthogonal.

Some properties of determinants thrown in for good measure.

$$|A| \neq 0 \text{ if } \underline{b}^T A \underline{b} \neq 0 \text{ for all } \underline{b} \neq \underline{0}$$

$$|A^T| = |A|$$

$$|A^{-1}| = |A|^{-1}$$

If $|AB| = |A||B|$

If P is orthogonal, $|P| = \underline{+}1$

$$1 = |I| = |P^T P| = |P^T||P| = |P|^2$$

$$|P| = \underline{+}1$$

The determinant of a matrix is the product of its eigenvalues:

$$A = P \Lambda P^{-1}$$

$$|A| = |P \Lambda P^{-1}|$$

$$= |P||\Lambda||P^{-1}|$$

$$= |P||P|^{-1}|\Lambda|$$

$$= |\Lambda|$$

$$= \lambda_1 \lambda_2 - - - \lambda_n$$

IV. Statistical State Estimation

Some Philosophical Remarks on the Nature of the Problem

It is worth noting that from a physical measurement point of view, such quantities as the position and velocity of a particle are not available to direct observation. They are abstract mathematical constructs, evolving according to an empirically developed law and should not be confused with "real" aspects of the system. They are useful only in so far as they serve to predict the observables of the system, i.e. scalar quantities such as distance, angular measure and speed; the only aspects of the system to which the measurement process has direct access.

Mechanics considers a system "noise free" if it is possible to argue that, in principle, all that is needed for a perfect state determination is a perfect sensor. In addition, the assumption is usually made in solving particular problems that the "equations of motion" are perfectly known. In most engineering problems, neither of these assumptions is true. Even were they true, a further practical difficulty arises in the attempt to find the "state" variables as a function of the observables, as the observables are generally transcendental functions of the state variables and time, and therefore not amenable to algebraic solution.

The problem in state estimation is therefore two-fold:

1. Circumvent the mathematical difficulties associated with determination of the system state in terms of the observables, and (2) do so in a manner with reflects the imperfect character of the measurements and knowledge of the system law of motion. In order to do this, the attitude will be adopted that the measuring device is an inseparable part of the system in the dynamic sense: its properties effect the determination of the system state.

The system under study will be taken as two particles in orbit around a planet, between which measurements of range, range rate, and relative direction may be made with an appropriate sensor. Any biases which may effect a sensor measurement are classified as state variables. Otherwise, all noise on the system (state as well as sensor) will be considered as random with zero mean. No assumption about its statistical distribution will be made (Fig. I). Also, for purposes of specific treatment, a measurement coordinate frame is depicted in Fig. II:

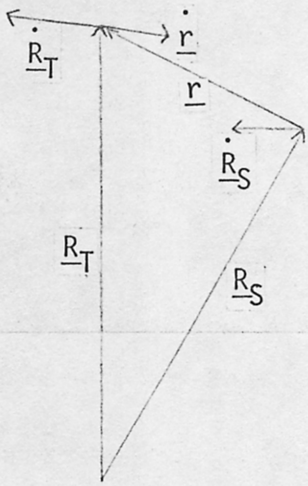


FIGURE I

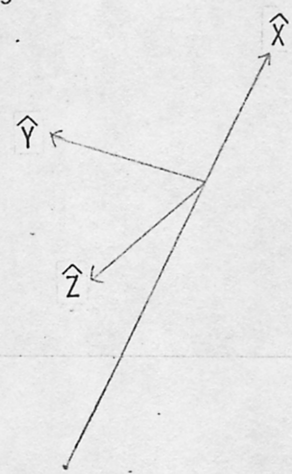


FIGURE II

$\underline{R}_S$ = spacecraft position vector

$\dot{\underline{R}}_S$ = spacecraft velocity vector

$\underline{R}_T$ = target position vector

$\dot{\underline{R}}_T$ = target velocity vector

$\underline{r}$ = relative position vector

$\dot{\underline{r}}$ = relative velocity vector

$\hat{X}$ = unit $(\underline{R}_S)$

$\hat{Y}$ = unit $\underline{R}_S \times (\underline{R}_S \times \dot{\underline{R}}_S)$

$\hat{Z}$ = unit $(\underline{R}_S \times \dot{\underline{R}}_S)$

Total state of system

$$X = \begin{bmatrix} \underline{R}_S \\ \dot{\underline{R}}_S \\ \underline{R}_T \\ \dot{\underline{R}}_T \\ \underline{\beta} \end{bmatrix}$$

$\underline{\beta}$ estimated constants

In order to solve for the state in terms of observable quantities, a method of differential corrections will be devised. Consider an observable of the system, Q

$$Q = Q\ (\underline{X})$$

and its expansion about a reference state, $\underline{X}_R$:

$$Q(\underline{X}) = Q(\underline{X}_R) + \frac{\partial Q}{\partial \underline{X}}\bigg|_{\underline{X}_R}^{T} (\underline{X}-\underline{X}_R) + (\underline{X}-\underline{X}_R)^T \left\{ \frac{\partial}{\partial \underline{X}} \left(\frac{\partial Q}{\partial \underline{X}}\right)^T \right\}_{\underline{X}_R}^{T} (\underline{X}-\underline{X}_R) + \text{--}$$

assume that $\underline{X}_R$ is chosen sufficiently close to $\underline{X}$ that derivative terms higher than first are neglible:

$$Q(\underline{X}) = Q(\underline{X}_R) + \frac{\partial Q}{\partial \underline{X}}\bigg|_{\underline{X}_R}^{T} (\underline{X}-\underline{X}_R)$$

defining $\delta Q = Q(\underline{X}) - Q(\underline{X}_R)$, $\delta \underline{X} = (\underline{X}-\underline{X}_R)$

4.1 $$\delta Q = \frac{\partial Q}{\partial \underline{X}}\bigg|_{\underline{X}_R}^{T} \delta \underline{X}$$

Conventional usage will now be followed and such variations will be considered as truncated expansions, whenever the notation $\delta(\ \ )$ is used. The quantity

$$\frac{\partial Q}{\partial \underline{X}}\bigg|_{\underline{X}_R},$$

is called the geometry vector or mapping vector and will henceforth be denoted

$$\underline{b}_Q = \frac{\partial Q}{\partial \underline{X}}\bigg|_{\underline{X}_R} ; \quad \underline{b}_Q \quad (n\mathrm{x}1)$$

so that 4.1 becomes

4.2 $$\delta Q = \underline{b}_Q^T\, \delta \underline{X}$$

Equation 4.2 is the desired linear relationship between the observable Q and the state $\underline{X}$. As there are n elements of $\delta \underline{X}$ to determine, n independent measurements, δQ, will be required. Since these cannot normally be simultaneously obtained, it will be necessary to investigate the time history of $\delta \underline{X}$. For gravitational field, the derivative, $\dot{\underline{X}}$ is a vector function of $\underline{X}$:

$$4.3 \qquad \frac{d}{dt}\underline{X} = \underline{f}(\underline{X}) \qquad \underline{f}_{n\times 1}$$

for example, in the Keplerian case with $\underline{X}$ as previously defined

$$4.4 \qquad \frac{d}{dt}\underline{X} = \frac{d}{dt}\begin{bmatrix} \underline{R}_S \\ \dot{\underline{R}}_S \\ \underline{R}_T \\ \dot{\underline{R}}_T \\ \underline{\beta} \end{bmatrix} = \begin{bmatrix} \emptyset & I & \emptyset & \emptyset & \emptyset \\ -\frac{\mu}{R_S^3} I & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & I & \emptyset \\ \emptyset & \emptyset & -\frac{\mu}{R_T^3} I & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{bmatrix} \begin{bmatrix} \underline{R}_S \\ \dot{\underline{R}}_S \\ \underline{R}_T \\ \dot{\underline{R}}_T \\ \underline{\beta} \end{bmatrix} \qquad \begin{aligned} \emptyset &= [0]_{3\times 3} \\ I &= I_{3\times 3} \end{aligned}$$

If $\underline{X}$ and its derivatives with respect to state elements and time are continuous, the operations $\frac{d}{dt}$ and $\delta(\ )$ are interchangeable, i.e.

$$\frac{d}{dt}\delta(\ ) = \delta\frac{d}{dt}(\ )$$

Taking the variation of 4.3

$$4.5a \qquad \delta\left(\frac{d}{dt}\underline{X}\right) = \frac{d}{dt}\delta\underline{X} = \delta(\underline{f}(\underline{X})) = \left\{\frac{\partial}{\partial \underline{X}}\underline{f}^T\right\}\delta\underline{X} = \underline{F}\delta\underline{X}$$

For the Keplerian field of 4.4

$$F = \left\{ \frac{\partial}{\partial \underline{X}} \underline{f}^T \right\}^T = \begin{bmatrix} \emptyset & I & \emptyset & \emptyset & \emptyset \\ [G]_S & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & I & \emptyset \\ \emptyset & \emptyset & [G]_T & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{bmatrix}_{n \times n} \qquad \begin{aligned} [G]_S &= \frac{\mu}{\underline{R}_S^5} (3\underline{R}_S\underline{R}_S^T - R_S^2 I) \\ [G]_T &= \frac{\mu}{\underline{R}_T^5} (3\underline{R}_T\underline{R}_T^T - R_T^2 I) \end{aligned}$$

Therefore

4.5b $$\delta \dot{\underline{X}} = F \delta \underline{X}$$

This equation may be integrated numerically. A more useful solution takes the form of

4.6 $$\delta \underline{X}(t) = \Phi(t, t_o)\, \delta \underline{X}(t_o) \qquad \Phi_{nxn}$$

Φ is called the state transition matrix. Differentiating 4.6 and substituting for $\delta \underline{X}$ and $\delta \dot{\underline{X}}$ in 4.5b:

$$\delta \dot{\underline{X}} = \dot{\Phi} \delta \underline{X}(t_o) = F \, \Phi \, \delta \underline{X}_o$$

By inspection then

4.7 $$\dot{\Phi} = F\Phi, \text{ subject to } \Phi(t_o, t_o) = I$$

For a noise-free system, the problem is now solved. A deviation at t can be referenced to any other time. Suppose a set of n measurements has been made times t_1, t_2 - - t_n.

$$\delta Q_1 = \underline{b}_Q^T(t_1)\ \delta \underline{X}(t_1) = \underline{b}_Q^T(t_1)\,\Phi\,(t_1, t_n)\,\delta \underline{X}_n$$

$$\delta Q_2 = \underline{b}_Q^T(t_2)\ \delta \underline{X}(t_2) = \underline{b}_Q^T(t_2)\ \Phi\ (t_2, t_n)\,\delta \underline{X}_n$$

$$\begin{array}{cc} - & - \\ - & - \end{array}$$

4.8a

$$\delta Q_n = \underline{b}^T{}_Q(t_n)\ \delta \underline{X}(t_n) = \underline{b}_Q^T(t_n)\ \Phi\ (t_n, t_n)\ \delta \underline{X}_n$$

Let $\underline{h}^T{}_i = \underline{b}_Q^T\ \Phi(t_i, t_n)$, then

4.8b

$$\begin{bmatrix} \delta Q_1 \\ \delta Q_2 \\ - \\ - \\ \delta Q_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} \underline{h}_1^T \\ \underline{h}_2^T \\ - \\ - \\ \underline{h}_n^T \end{bmatrix}_{n\times n} \delta \underline{X}_n(t_n)$$

4.9a

$$\delta \underline{X}(t_n) = \begin{bmatrix} \underline{h}_1^T \\ \underline{h}_2^T \\ - \\ - \\ \underline{h}_n^T \end{bmatrix}^{-1} \begin{bmatrix} \delta Q_1 \\ \delta Q_2 \\ - \\ - \\ \delta Q_n \end{bmatrix}$$

If more than n measurements are made, say m (>n), the solution has the form

$$\delta \underline{X}(t_m) = \left\{ \left[\underline{h}_i^T\right]^T \left[\underline{h}_i^T\right] \right\}^{-1} \left[\underline{h}_i^T\right]^T \left[\delta Q_i\right]$$

4.9b

$$= \left\{ \begin{bmatrix} \underline{h}_2^T \\ - \\ - \\ \underline{h}_m^T \end{bmatrix} \cdot \begin{bmatrix} \underline{h}_2^T \\ - \\ - \\ \underline{h}_m^T \end{bmatrix} \right\}^{-1} \begin{bmatrix} \underline{h}_2^T \\ - \\ - \\ \underline{h}_m^T \end{bmatrix} \begin{bmatrix} \delta Q_2 \\ - \\ - \\ \delta Q_m \end{bmatrix}$$

Equation 4.9a is called the deterministic solution of $\delta\underline{X}(t_n)$, 4.9b is the unweighted least squares solution of $\delta\underline{X}(t_n)$. If statistical information on the relative quality of various measurements is available, a more virtuous estimate might be obtained by multiplying each side of 4.8 by the relative weights; customarily used is the covariance of measurement noise for the m measurements, C :

4.9c $\quad \delta\underline{X}(t_n) = \left[H^T[C]H\right]^{-1} \; H^T[C]\,\overrightarrow{\delta Q} \qquad H = \begin{bmatrix} \underline{h}_1^T \\ \underline{h}_2^T \\ - \\ - \\ \underline{h}_n^T \end{bmatrix}$

This is called the weighted least squares estimate of $\delta X(t_n)$. An estimator of the type 4.9c is also called a batch estimator. Normally, a set of data $\overrightarrow{\delta Q}$ will be processed several times, each time using the estimate, $\delta\underline{X}$, to compute a new $\underline{X}_R^*$:

$$\underline{X}_R^* = \underline{X}_R + \delta\,\underline{X}$$

whereupon new values of $Q(\underline{X}_R)$ and hence $\delta Q(t_i)$, $\underline{b}_Q(t_i)$ and $\Phi(t_i, t_n)$ are computed and used in the next pass. $\delta\underline{X}$ converges to near zero and the assumption that higher order derivatives are negligible is accurately full-filled. A number of defects with the technique make it unsuitable for

many applications: all data from a set of measurements must be stored, as well as the H matrix, which may become quite large; a large matrix inversion is required which is slow and sometimes numerically difficult or inaccurate; in most cases, several "passes" are needed for complete convergence. For these reasons, a technique of sequential estimation has been developed, which avoids these problems, as will be seen. First, an investigation into the propagation of state errors is required.

Let $\underline{X}_E$ be an estimate of X. Then

$$\delta \underline{X}_E = \underline{X}_E - \underline{X}_R$$

Since it has been found that

$$\delta \underline{X}(t) = \Phi\, \delta \underline{X}(t_o)$$

then

$$\delta \underline{X}_E(t) = \Phi\, \delta \underline{X}_E(t_o)$$

and

$$\underline{e}(t) = \delta \underline{X}_E(t) - \delta \underline{X}(t) = \Phi \left[\delta \underline{X}_E(t_o) - \delta \underline{X}(t_o) \right]$$
$$= \Phi\, \underline{e}_o$$

Furthermore

4.10 $$E(t) = \overline{\underline{e}(t)\underline{e}(t)^T} = \overline{\Phi \underline{e}(t_o)\underline{e}(t_o)^T \Phi^T} = \Phi E(t_o) \Phi^T$$

Demonstrating the evolution of the covariance of state errors. An nxn symmetric matrix may be expressed as the sum of n linearly independent forms of the kind

$$E = \frac{1}{n}\left[\underline{e}_1\underline{e}_1^T + \underline{e}_2\underline{e}_2^T = - - + \underline{e}_n\underline{e}_n^T\right] = \frac{1}{\sqrt{n}}\left[\underline{e}_1\ \underline{e}_2 - - -\ \underline{e}_n\right]\begin{bmatrix}\underline{e}_1^T \\ \underline{e}_2^T \\ - \\ - \\ \underline{e}_n^T\end{bmatrix}\frac{1}{\sqrt{n}}$$

Define $W = \frac{1}{\sqrt{n}}\left[\underline{e}_1\ \underline{e}_2 - -\ \underline{e}_n\right]$ and thus

$$E = WW^T$$

By inspection of 4.10,

$$E(t) = W(t)W(t)^T = \Phi E(t_o)\Phi^T = \Phi W(t_o)W(t_o)^T\Phi^T$$

4.11 $$W(t) = \Phi W(t_o)$$

Physically, W is a set of linearly independent vectors drawn from the sample space of E which have the property that

$$\frac{1}{n}\sum_{i=1}^{n}\underline{e}_i\underline{e}_i^T = \int_V \underline{\xi}\ \underline{\xi}^T f(\underline{\xi})\ d\xi_1\ d\xi_2 - - d\xi_n$$

That is, a finite, discrete sample from a continuous distribution, having the particular property that their simple average produces the covariance defined by the distribution function $f(\xi)$. They "typify" the distribution.

Sequential Estimator (Kalman Filter)

The sequential estimator has the form

4.12 $$\delta\hat{\underline{X}}(t_n) = \delta\hat{\underline{X}}'(t_n) + \underline{w}_n\left(\delta\tilde{Q}(t_n) - \delta\hat{Q}'(t_n)\right)$$

where

$$\delta \widehat{\underline{X}}'(t_n) = \Phi(t_n, t_{n-1})\ \delta \widehat{\underline{X}}(t_{n-1})$$

$$\widehat{\delta Q}' = \underline{b}_Q^T(t_n)\ \delta \widehat{\underline{X}}'$$

$$\delta \tilde{Q} = \tilde{Q} - Q(\underline{X}_R)$$

$$\tilde{Q} = \text{measured value of } Q$$

$\underline{w}$ an nx1 vector to be determined

Equation 4.12 proposes that if $\delta \widehat{\underline{X}}'$ is an estimate of $\delta \underline{X}$, a better estimate of $\delta \underline{X}$ is the liner combination of $\delta \widehat{\underline{X}}'$ with a weighting vector, $\underline{w}$, multiplied by the difference between the measured value of δQ ($\delta \tilde{Q}$) and the expected value of δQ ($\widehat{\delta Q}'$) based on $\delta \widehat{\underline{X}}'$. To determine $\underline{w}$, it is necessary to consider what constitutes a "better" estimate of $\delta \underline{X}$. Clearly, in a particular circumstance, no direct knowledge of the existing error is possible, and it will be necessary to deal with some average function of the error. Since in general the state error distribution is unknown, one must work with the mean error and the covariance of errors. Minimizing the average squared error seems to be the most physically meaningful objective. To this end, use 4.12 to construct an expression for the state error:

$$\underline{e}(t_n) = \delta \underline{X}_n - \delta \widehat{\underline{X}}_n = \delta \underline{X}_n - \delta \widehat{\underline{X}}'_n - \underline{w}_n(\delta \tilde{Q}_n - \delta Q'_n)$$

$$= \widehat{\underline{e}}'_n - \underline{w}_n(\underline{b}_Q^T\ \delta \underline{X} + \alpha_Q - \underline{b}_Q^T\ \delta \widehat{\underline{X}}'_n)$$

$$= \widehat{\underline{e}}'_n - \underline{w}_n \underline{b}_Q^T\ (\delta \underline{X} - \delta \widehat{\underline{X}}'_n) - \alpha_Q \underline{w}_n$$

$$= \widehat{\underline{e}}'_n - \underline{w}_n \underline{b}_Q^T\ \underline{e}'_n - \alpha_Q \underline{w}_n$$

4.13 $$= (I - \underline{w} \underline{b}_Q^T)\ \underline{e}'_n - \alpha_Q \underline{w}_n$$

Where

$$\tilde{\delta Q}' = \tilde{Q}(\underline{X}) - Q(\underline{X}_R)$$

$$= Q(\underline{X}) + \alpha_Q - Q(\underline{X}_R)$$

$$= \text{measured value of } \delta Q$$

and

$$\alpha_Q = \text{random measurement noise}$$

By direct calculation, the covariance of state errors E is

$$E = \overline{\underline{e}\underline{e}^T} = \overline{(I - \underline{w}\underline{b}_Q^T)\, \underline{e}_n' \underline{e}_n^T (I - \underline{w}\underline{b}_Q^T)^T + \alpha_Q^2\, \underline{w}\underline{w}^T}$$
$$- \overline{(I - \underline{w}\underline{b}_Q^T)\, \underline{e}_n \underline{w}'^T \alpha_Q - \alpha_Q \underline{w}\underline{e}_n' (I - \underline{b}_Q \underline{w}^T)^T} \quad (4.14a)$$

Since previous state errors are uncorrelated with current measurement errors, the terms involving

$$\overline{\underline{e}_n'\alpha_Q}, \qquad \overline{\alpha_Q\underline{e}_n^{T\,\prime}}$$

average to zero. Hence the expression for the covariance is

4.14b $$E = (I - \underline{w}\,\underline{b}_Q^T)\,E_r'\,(I - \underline{w}\,\underline{b}_Q^T)^T + \overline{\alpha_Q^2}\,\underline{w}\,\underline{w}^T$$

The diagonal elements of E, E' are the respective average squared errors in the components of $\underline{X}_E$. Hence, by calculus of variations $\underline{w}$ will be chosen so as to minimize the elements of E, including its diagonal: For an extremum of E with respect to $\underline{w}$

$$\delta E = \left[(- \delta\underline{w}\,\underline{b}_Q^T E'\ (I - \underline{w}\,\underline{b}_Q^T)\right] + \overline{\alpha_Q^2}\,\delta\underline{w}\,\underline{w}^T$$

$$+ \left[(I - \underline{w}\,\underline{b}_Q^T)^T E'\ \underline{b}_Q\ (-\delta\underline{w}^T)\right] + \overline{\alpha_Q^2}\,\underline{w}\delta\underline{w}^T$$

$$= \left\{ \left[-E'\underline{b}_Q + (\underline{b}_Q^T E'\ \underline{b}_Q + \overline{\alpha_Q^2})\ \underline{w}\right]\delta\,\underline{w}^T \right\}^T + \left[-E'\underline{b}_Q + (\underline{b}_Q^T E_n'\underline{b}_Q + \overline{\alpha_Q^2})\ \underline{w}\right]\delta\underline{w}^T$$

$$= [0] \quad \text{for an extremum}$$

This condition will apparently be met if

$$\left[-E'\underline{b}_Q + (\underline{b}_Q^T\ E'\underline{b}_Q + \overline{\alpha_Q^2})\ \underline{w}\right] = \underline{0}$$

which implies

4.15 $$\underline{w} = \frac{E'\underline{b}_Q}{\overline{\alpha_Q^2} + \underline{b}^T E'\underline{b}}$$

This is the Kalman Filter. It remains to show that extremum of E for which $\underline{w}$ has this value is a minimum:

Using the expression for δE:

$$\delta^2 E = (\underline{b}_Q^T\, E'\underline{b}_Q + \overline{\alpha_Q^2})(\delta\underline{w}\delta\underline{w}^T + \delta\underline{w}\delta\underline{w}^T)$$

It is clearly apparent that

$$(\underline{b}_Q^T\, E'\underline{b}_Q + \overline{\alpha_Q^2}) > 0$$

always since E' is positive definite. Further more, the diagonal terms of $\delta\underline{w}\ \delta\underline{w}^T$ are positive for all real variations, $\delta\underline{w}$. The corresponding second variations of E, those of the mean squared error, are therefore positive and minimum. Figure III presents an operational flow chart of the filter.

$\underline{X}_E(t)$, $\underline{X}_E(t)$, $E(t_o)$

$$E'(t) = \Phi(t,t_o)E(t_o)\Phi^T(t,t_o)$$ update covariance to present time

$$\underline{b}_Q = \frac{\partial Q}{\partial \underline{X}_R}(t)$$ compute geometry vector

$$\underline{w} = \frac{E'(t)\ \underline{b}_Q}{\overline{\alpha_Q^2} + \underline{b}^T E'\underline{b}}$$ compute weighting vector

$$\widehat{\delta\underline{X}}' = \underline{X}'_E - \underline{X}_R$$ compute estimated deviation

$$\widehat{\delta Q}' = \underline{b}^T\widehat{\delta\underline{X}}'$$ compute estimated δQ

$$\delta\tilde{Q} = \tilde{Q} - Q(\underline{X}_R)$$ compute measured δQ

$$\delta\hat{\underline{X}} = \delta\hat{X}' + \underline{w}(\delta\tilde{Q} - \delta\hat{Q}')$$ compute update to estimates $\delta\underline{X}$

$$\underline{X}_E = \underline{X}_E' + \delta\hat{X}$$ update $\underline{X}_E$

$$E = (I - \underline{w}\,\underline{b}_Q^T)E'(I - \underline{w}\,\underline{b}_Q^T)^T + \underline{w}\,\underline{w}^T\,\overline{\alpha^2}_Q$$ update E

Figure III.

V. Further development and insights.

Propagation of Errors

For a Keplerian force field, the state transition matrix, which is a solution to the equation.

$$\dot{\Phi}(t,t_o) = [F]\Phi(t,t_o) \qquad \Phi(t_o,t_o) = I$$

has the property that

$$\Phi^T J\,\Phi = J \qquad J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \qquad \begin{matrix} J^T J = I \\ J^2 = -I \end{matrix}$$

For as single vehicle, Φ is 6x6 and from 4.5b

$$\underline{F} = \begin{bmatrix} \phi & I \\ G & 0 \end{bmatrix} \qquad G = \frac{\mu}{R^5}\,(3\underline{R}\,\underline{R}^T - R^2 I)$$

To prove this property, note that at t_o

$$\Phi^T J \Phi = \underline{I}\;\underline{J}\;\underline{I} = \underline{J} \qquad \Phi(t_o,t_o) = \underline{I}$$

Evaluating

$$\frac{d}{dt}(\Phi^T \underline{J}\Phi) = \dot{\Phi}^T \underline{J}\,\Phi + \Phi^T \underline{J}\dot{\Phi}$$

$$= \Phi^T F^T \underline{J}\,\Phi + \Phi^T \underline{J} F \Phi$$

$$= \Phi^T \left[\underline{F}^T \underline{J} + \underline{J}\ \underline{F}\right] \Phi$$

$$\underline{F}^T \underline{J} = \begin{bmatrix} 0 & I \\ G & 0 \end{bmatrix} \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} = \begin{bmatrix} -I & 0 \\ 0 & G \end{bmatrix} = -\underline{J}F$$

Therefore $\underline{F}^T \underline{J} + \underline{J}\ \underline{F} = [0]$ and

$$\frac{d}{dt}(\Phi^T J \Phi) = [0] \Rightarrow \Phi^T J \Phi = \text{constant} = J$$

Now note that $|\underline{J}| = 1$ and therefore

$$\left|\Phi^T \Phi \underline{J}\right| = \left|\Phi^T\right| \left|\underline{J}\right| \left|\Phi\right|$$

$$= \left|\Phi\right|^2 1$$

$$= \left|J\right| = 1$$

For a Keplerian field the propagation of E is described by

$$E(t) = \Phi\, E(t_0)\, \Phi^T$$

and its determinant by

$$\begin{aligned}|\underline{E}(t)| &= |\Phi\, E(t_o)\, \Phi^T| \\ &= |\Phi|^2\, |E(t_o)| \\ &= |E(t_o)|\end{aligned}$$

The physical interpretation of this fact is taken to be that the total "amount" of error in the system remains bounded in time for a Keplerian force field.

Practical Computational Techniques

It usually happens that operations performed on E by virtue of equation 4.10 or 4.14b will cause it to be no longer symmetric due to computational inaccuracies. Should this happen, it no longer represents a covariance matrix and something must be done. One method is to perform the replacement operation

$$E = \frac{1}{2}(E + E^T)$$

which will symmetrize E in any digital computer worthy of the name. An alternative approach, which offers considerable other computational advantages will be constructed. Making use of the fact that any covariance matrix may be represented as a linear combination of product transposes, define

$$\begin{aligned}E' &= \frac{1}{N}\left[\underline{e}_1\underline{e}_1^T + \underline{e}_2\underline{e}_2^T - - - \underline{e}_N\underline{e}_N^T\right] \\ &= \frac{1}{\sqrt{N}}\left[\underline{e}_1\ \underline{e}_2\ - -\ \underline{e}_N\right] \qquad \left[\underline{e}_1\ \underline{e}_2\ - -\ \underline{e}_N\right]^T \frac{1}{\sqrt{N}} \\ &= W'W'^T\end{aligned}$$

$$W = \frac{1}{\sqrt{N}}\left[\underline{e}_1\ \underline{e}_2\ - -\ \underline{e}_N\right]$$

Let

$$\underline{z}_Q = W'^T \underline{b}_Q$$

and rewrite equation 4.15 as

5.1 $$\underline{W} = \left(\frac{W' \underline{z}_Q}{\overline{\alpha_Q^2} + \underline{z}_Q^T \underline{z}}\right)$$

Equation 4.14b, the update equation for E, requires careful consideration:

Substituting for $\underline{w}$:

$$WW^T = \left(I - \frac{W'\underline{z}\ \underline{b}^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right) W'W'^T \left(I - \frac{W'\underline{z}\ \underline{b}^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right)^T + \frac{W'\underline{z}\ \underline{z}^T W'^T}{(\overline{\alpha_Q^2} + \underline{z}^T\underline{z})^2}\ \overline{\alpha_Q^2}$$

$$= \left(W' - \frac{W'\underline{z}\ \underline{z}^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right)\left(W'^T - \frac{\underline{z}\ \underline{z}^T W'^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right) + \frac{W'\underline{z}\ \underline{z}^T W'^T}{(\overline{\alpha_Q^2} + \underline{z}^T\underline{z})^2}\ \overline{\alpha_Q^2}$$

$$= W'\left[\left(I - \frac{\underline{z}\ \underline{z}^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right)\left(I - \frac{\underline{z}\ \underline{z}^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right) + \frac{\underline{z}\ \underline{z}^T\ \overline{\alpha_Q^2}}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}}\right] W'^T$$

$$= W'\left[\left(I - \frac{2\underline{z}\ \underline{z}^T}{\overline{\alpha_Q^2} + \underline{z}^T\underline{z}} + \frac{\underline{z}\ \underline{z}^T(\underline{z}^T\underline{z})}{(\overline{\alpha_Q^2} + \underline{z}^T\underline{z})^2} + \frac{\underline{z}\ \underline{z}^T\ (\overline{\alpha_Q^2}}{(\overline{\alpha_Q^2} + \underline{z}^T\underline{z})^2}\right)\right] W'^T$$

$$= W' \left(I - \frac{\underline{z}\,\underline{z}^T}{\overline{\alpha}_Q^2 + \underline{z}^T\underline{z}} \right) W'^T$$

Look for a matrix of the form

$$\left(I - \frac{K\,\underline{z}\,\underline{z}^T}{\overline{\alpha}_Q^2 + \underline{z}^T\underline{z}} \right)$$

such that

$$\left(I - \frac{K\underline{z}\,\underline{z}^T}{\overline{\alpha}_Q^2 + \underline{z}^T\underline{z}} \right)^2 = \left(I - \frac{\underline{z}\,\underline{z}^T}{\overline{\alpha}_Q^2 + \underline{z}^T\underline{z}} \right)$$

$$I - \frac{2K\underline{z}\,\underline{z}^T}{\overline{\alpha}_Q^2 + \underline{z}^T\underline{z}} + \frac{(K^2\underline{z}^T\underline{z})\,\underline{z}\,\underline{z}^T}{(\overline{\alpha}_Q^2 + \underline{z}^T\underline{z})^2} = I - \frac{\underline{z}\,\underline{z}^T}{\overline{\alpha}_Q^2 + \underline{z}^T\underline{z}}$$

$$\left(-\frac{2K}{a} + \frac{1}{a} + \frac{K^2\underline{z}^T\underline{z}}{a^2}\right) \underline{z}\,\underline{z}^T = [0]$$

$$K^2 - \frac{2a}{\underline{z}^T\underline{z}} K + \frac{a}{\underline{z}^T\underline{z}} = 0$$

$$K = \frac{\dfrac{2\,a}{\underline{z}^T\underline{z}} - \sqrt{\dfrac{4a^2}{(\underline{z}^T\underline{z})^2} - \dfrac{4a}{\underline{z}^T\underline{z}}}}{2}$$

$$= \frac{a}{\underline{z}^T\underline{z}}\left(1 - \sqrt{1 - \frac{\underline{z}^T\underline{z}}{a}}\right)$$

where $a = \underline{z}^T\underline{z} + \overline{\alpha_Q^2}$. This may be simplified by letting

$$c = \frac{1}{K} = \frac{\dfrac{\underline{z}^T\underline{z}}{a}}{\left(1 + \sqrt{1 - \dfrac{\underline{z}^T\underline{z}}{a}}\right)} = \frac{\dfrac{\underline{z}^T\underline{z}}{a}\left(1 + \sqrt{1 - \dfrac{\underline{z}^T\underline{z}}{a}}\right)}{(1)^2 - \left(\sqrt{1 - \dfrac{\underline{z}^T\underline{z}}{a}}\right)^2}$$

$$= 1 + \sqrt{1 - \frac{\underline{z}^T\underline{z}}{a}}$$

$$= 1 + \sqrt{\frac{\overline{\alpha_Q^2}}{a}}$$

5.2

Putting 1/c in for K

$$WW^T = W'\left(I - \frac{\underline{z}\,\underline{z}^T}{a}\right) W'^T$$

$$= W'\left(I - \frac{\underline{z}\,\underline{z}^T}{ca}\right)^2 W'^T$$

Therefore

5.3 $$W = W'\left(I - \frac{\underline{z}\,\underline{z}^T}{ca}\right)$$

The advantages to working with W rather than E are several:

(1) Since $WW^T = E$, any function of W of the form $\underline{z}^T\underline{z}$ or $W\underline{z}$ implies a symmetric (positive definite) E.

(2) The updating equation for W is simpler than that for E and involves fewer computations.

(3) Since W is a construction of representative error vectors $\{\underline{e}_i\}$, its propagation in time is more easily discussed.

Noting that $E(t) = \Phi(t,t_o)\, E_o\, \Phi^T(t,t_o)$, it is found that

$$E(t) = W(t)W(t) = \Phi\, W_o\, W_o^T\, \Phi^T$$

5.4a $$W(t) = \Phi(t,t_o)\, W(t_o)$$

Also

$$\dot{W}(t) = \dot{\Phi}\, W_o = F\, \Phi\, W_o$$

5.4b $$= FW$$

A third method of advancing W is to note that by definition

$$\underline{e}_i(t_o) = \underline{X}_E^i(t_o) - \underline{X}(t_o) \qquad \underline{X}_E = \text{estimated state}$$

$$\underline{X} = \text{actual state}$$

therefore

$$\underline{e}_i(t) = \underline{X}_E^i(t) - \underline{X}(t)$$

Let

$$\underline{X}_E^i(t_o) = \underline{X}_E(t_o) + \underline{e}_i(t_o)$$

$$\underline{e}_i(t) = \underline{X}_E^i(t) - \underline{X}_E(t)$$

where $\underline{X}_E^i(t)$ and $\underline{X}_E(t)$ are the respective vectors at to integrated to time t. This constitutes a more exact solution to the problem of propagating E and makes no linearizing assumptions as were made to obtain 4.5b, which is the basis for equations 5.4a and 5.4b.

Definition of the Reference State

For practical purposes, the reference state is generally taken to be the estimated state at each instant. In this case

$$\delta \widehat{\underline{X}'} = \widehat{\underline{X}'} - \underline{X}_R$$

$$= \widehat{\underline{X}'} - \widehat{\underline{X}'} = \underline{0}$$

$$\delta \widehat{Q'} = \underline{b}_Q^T \; \delta \widehat{\underline{X}'} = 0$$

$$\delta \tilde{Q} = \tilde{Q} - Q\ (\underline{X}_R)$$

$$= \tilde{Q} - Q\ (\widehat{\underline{X}'})$$

$$\delta \hat{\underline{X}} = \underline{w}\, \delta \tilde{Q} = \underline{w}\ (\tilde{Q} - Q\ (\hat{\underline{X}}'))$$

$$\hat{\underline{X}} = \hat{\underline{X}}' + \delta \hat{\underline{X}}$$

This definition guarantees that as the differential correction process occurs, $\underline{X}_R$ is sufficiently close to $\underline{X}$ so that the linearizing assumptions of Sec. IV are valid.

Computation of the Geometry ($\underline{b}$) Vecter

For a range measurement

$$r = \left[\underline{r}^T \underline{r}\right]^{1/2} = \left[(\underline{R}_T - \underline{R}_S)^T (\underline{R}_T - \underline{R}_S)\right]^{1/2}$$

$$\delta r = 1/2 \left[\underline{r}^T \underline{r}\right]^{-1/2} (\delta \underline{r}^T \underline{r}) + 1/2 \left[\underline{r}^T \underline{r}\right]^{-1/2} (\underline{r}^T \delta \underline{r}) + \beta_r$$

$$= \left[\underline{r}^T \underline{r}\right]^{-1/2} \underline{r}^T \delta \underline{r} + \beta_r$$

$$= \hat{\underline{r}}^T \delta \underline{r} = \hat{\underline{r}}^T \left[\delta \underline{X}_T - \delta \underline{X}_s\right] + \beta_r$$

Where β_r is the range measurement bias, and $\hat{\underline{r}}$ = unit $(\underline{r})$. Therefore,

$$\underline{b}_r = \begin{bmatrix} -\hat{\underline{r}} \\ \underline{0} \\ \hat{\underline{r}} \\ \underline{0} \\ \underline{k}_r \end{bmatrix} \quad \text{for } \underline{k}_r = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For a range-rate measurement

$$\dot{r} = \hat{\underline{r}}^T \dot{\underline{r}} = \frac{\underline{r}^T}{r} \dot{\underline{r}}$$

$$\delta \dot{r} = \frac{\delta \underline{r}^T}{r} \dot{\underline{r}} - \frac{\underline{r}^T \dot{\underline{r}}}{r^2} \delta r + \frac{\underline{r}^T}{r} \delta \dot{\underline{r}} + \beta_{\dot{r}}$$

$$= \frac{\delta \underline{r}^T}{r} \dot{\underline{r}} - \frac{\underline{r}^T \dot{\underline{r}}}{r^2} (\hat{\underline{r}} \cdot \delta \underline{r}) + \frac{\underline{r}^T}{r} \delta \dot{\underline{r}} + \beta_{\dot{r}}$$

$$= \frac{1}{r}\left[\dot{\underline{r}} - (\hat{\underline{r}}^T \dot{\underline{r}}) \hat{\underline{r}}\right]^T \delta \underline{r} + \hat{\underline{r}} \delta \dot{\underline{r}} + \beta_{\dot{r}}$$

$$= -\frac{1}{r}\left[\hat{\underline{r}} \times (\hat{\underline{r}} \times \dot{\underline{r}})\right]^T \delta \underline{r} + \hat{\underline{r}} \delta \dot{\underline{r}} + \beta_{\dot{r}}$$

The geometry vector for range-rate is accordingly

$$\underline{b}_{\dot{r}} = \begin{bmatrix} +\frac{1}{r}\left[\underline{\hat{r}} \times (\underline{\hat{r}} \times \underline{\dot{r}})\right] \\ -\underline{\hat{r}} \\ \frac{-1}{r}\left[\underline{\hat{r}} \times (\underline{\hat{r}} \times \underline{\dot{r}})\right] \\ +\underline{\hat{r}} \\ \underline{k}_{\dot{r}} \end{bmatrix} \qquad \underline{k}_{r} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

For azimuth and altitude measurements, defined according to Fig. II as

$$AZ = TAN^{-1} \left(\frac{\underline{\hat{z}} \cdot \underline{\hat{r}}}{\hat{Y} \cdot \underline{\hat{r}}}\right)$$

$$EL = SIN^{-1} (\underline{\hat{x}} \cdot \underline{\hat{r}})$$

Some tedious algebraic hack produces

$$\underline{b}_{AZ} = \begin{bmatrix} -(\hat{x} \times \underline{r}) / |\hat{x} \times \underline{r}|^2 \\ \underline{0} \\ (\underline{\hat{x}} \times \underline{\hat{r}}) / |\underline{\hat{x}} \times \underline{r}|^2 \\ \underline{0} \\ \underline{k}_{AZ} \end{bmatrix} \qquad \underline{k}_{AZ} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{b}_{EL} = \begin{bmatrix} -\frac{|\hat{\underline{x}} \times \hat{\underline{r}}|}{r} (\hat{\underline{r}} \times \underline{b}_{AZ}) \\ \underline{0} \\ \frac{|\hat{\underline{x}} \times \hat{\underline{r}}|}{r} (\hat{\underline{r}} \times \underline{b}_{AZ}) \\ \underline{0} \\ \underline{k}_{EL} \end{bmatrix} \qquad \underline{k}_{EL} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

For these specific cases, the bias sub-vector of the state is defined as

$$\underline{\beta} = \begin{bmatrix} \beta r \\ \beta \dot{r} \\ \beta AZ \\ \beta EL \end{bmatrix}.$$

<u>Assorted comments about the weighting vector, $\underline{w}$.</u>

$$\underline{w} = \frac{E'\underline{b}}{\overline{\alpha_Q^2} + \underline{b}^T \underline{E}' \underline{b}} = \frac{W'\underline{z}}{\overline{\alpha_Q^2} + \underline{z}^T \underline{z}}$$

Examine $\underline{b}^T \underline{E}' \underline{b}$:

$$\underline{b}^T \underline{E}' \underline{b} = \underline{b}^T \; \overline{E'E'^T} \underline{b}$$

$$= \overline{\underline{b}^T \underline{e}' \underline{e}^{T'} \underline{b}}$$

$$= \overline{U_Q^{2'}}$$

where $\underline{b}^T\underline{e} = U_Q$, the a' priori uncertainty in Q due to state errors. Since $\overline{\alpha_Q^2}$ is the variance of Q due to measurement noise, the term

$$\overline{\alpha_Q^2} + \underline{b}^T E' \underline{b} = \overline{\alpha_Q^2} + \underline{z}^T \underline{z}$$

$$= \overline{\alpha_Q^2} + \overline{U_Q^{2'}}$$

is the total variance of Q due to measurement noise and state uncertainty. Now compute the estimate $\delta\hat{Q}$, the estimate of δQ based on $\delta\underline{\hat{x}}$:

$$\delta\underline{\hat{X}} = \delta\underline{\hat{X}}' + \underline{w}\ (\delta\tilde{Q} - \delta\hat{Q}')$$

$$\delta\hat{Q} = \underline{b}^T\ \delta\underline{\hat{X}}' + \underline{b}^T\underline{w}\ (\delta\tilde{Q} - \delta\hat{Q}')$$

$$= \delta\hat{Q}' + \frac{\underline{b}^T W \underline{z}}{\overline{\alpha_Q^2} + \overline{U_Q^{2'}}}\ (\delta\tilde{Q} - \delta\hat{Q}')$$

$$= \delta\hat{Q}' + \frac{\underline{z}^T\underline{z}}{\overline{\alpha_Q^2} + \overline{U_Q^{2}}'}\ (\delta\tilde{Q} - \delta\hat{Q}')$$

$$= \delta\hat{Q}' + \frac{\overline{U_Q^2}}{\overline{\alpha_Q^2} + \overline{U_Q^{2'}}}\ (\delta\tilde{Q} - \delta\hat{Q}')$$

Finally

$$\delta\hat{Q} = \delta\hat{Q}' \left(\frac{\overline{\alpha_Q^2}}{\overline{\alpha_Q^2} + \overline{U_Q^{2'}}}\right) + \delta\tilde{Q} \left(\frac{\overline{U_Q^2}}{\overline{\alpha_Q^2} + \overline{U_Q^{2'}}}\right)$$

Let the ratio of measurement noise to state uncertainty be denoted

$$\rho' = \overline{\alpha_Q^2} \,/\, \overline{U_Q^{2'}}$$

so that

$$\delta\hat{Q} = \delta\hat{Q}' \left(\frac{\rho'}{1 + \rho'}\right) + \delta\tilde{Q} \left(\frac{1}{1 + \rho'}\right)$$

Obviously if measurement noise variance, $\overline{\alpha_Q^2}$, is very small compared to the state uncertainty of δQ, $\overline{U_Q^2}$, we have $\rho' \sim 0$ and

$\delta\hat{Q} \simeq \delta\tilde{Q}$ <u>estimate equal to measured</u>

On the other hand, if the measurement noise is large compared to the state uncertainty $1/\rho' \sim 0$ and

$\delta\hat{Q} \simeq \delta\hat{Q}'$ <u>estimate equal to previous estimate</u>

Now look at the reduction of the state uncertainty of δQ, $\overline{U_Q^2}$, resulting from a measurement. From 5.3

$$W = W' \left(I - \frac{\underline{z}\,\underline{z}^T}{ca}\right) \qquad a = \overline{\alpha_Q^2} + \overline{U_Q^2} \qquad c = 1 - \sqrt{\overline{\alpha_Q^2}/a}$$

$$\overline{U_Q^2} = \underline{z}^T\underline{z} = \underline{b}^T W W^T \underline{b}$$

$$= \underline{b}^T W' \left(I - \frac{\underline{z}\,\underline{z}^T}{ca}\right)\left(I - \frac{\underline{z}\,\underline{z}^T}{ca}\right) W'^T \underline{b}$$

$$= \underline{b}^T W' \left(I - \frac{\underline{z}\,\underline{z}^T}{a}\right) W'^T \underline{b}$$

by definition

$$= \underline{z}^T \left(I - \frac{\underline{z}\,\underline{z}^T}{a}\right) \underline{z}$$

$$= \overline{U_Q^2}' - \frac{(\overline{U_Q^2})^2}{a}$$

$$= \overline{U_Q^2}' \left(1 - \frac{\overline{U_Q^2}}{a}\right)$$

$$= \overline{U_Q^2}' \left(\frac{a - \overline{U_Q^2}}{a}\right)$$

$$= \overline{U_Q^2}' \left(\frac{\overline{\alpha_Q^2}}{\overline{\alpha_Q^2} + \overline{U_Q^2}}\right)$$

$$= \overline{U_Q^2}' \left(\frac{\rho'}{1+\rho'}\right)$$

Again, as noted before, if $\rho' \simeq 0$,

$$\overline{U_Q^2} \simeq 0$$

i.e. the state uncertainty of δQ is greatly reduced. If the measurement noise is large compared to the state uncertainty, $\rho'/(1 + \rho') \sim 1$ and

$$\overline{U_Q^2} \simeq \overline{U_Q^2}'$$

This tallies with the expectation that if the measurement process is very noisy compared to the state errors, the average state errors are little reduced by the fixes. The case of a range measurement is particularly illustrative:

The form

$$\overline{U_r^2} = Z^T Z = \underline{b}_r^T WW^T \underline{b}_r = \underline{b}_r^T E \underline{b}_r = \overline{\underline{b}_r^T \underline{e}\,\underline{e}^T \underline{b}_r}$$

is the average squared state error along the line of sight. Typically, a good range sensor has 1 at random noise of about 10 feet on a measurement, thus

$$\overline{\alpha^2{}_r} = 100 \text{ ft}^2$$

If as is typical, the line of sight relative state errors are on the order of 5000 ft, we have

$$\overline{U_r^2} = 2{:}5 \times 10^7 \text{ ft}^2$$

$$\rho \simeq 4 \times 10^{-6}$$

$$\rho/1 + \rho \simeq (1 - \rho) = \rho - \rho^2$$

$$\simeq \rho$$

$$= 4 \times 10^{-6}$$

Across a mark, therefore

$$\delta\hat{Q} = \delta\hat{Q}' \left(\frac{\rho'}{1 + \rho'} \right) + \delta\tilde{Q} \left(\frac{1}{1 + \rho'} \right)$$

$$\cong \delta\hat{Q}' \,(4 \times 10^{-6}) + \delta\tilde{Q}\,(1 - 4 \times 10^{-6})$$

$$= \delta\hat{Q}' \,(.000004) + \delta\tilde{Q}\,(.999996)$$

The state uncertainty is reduced to

$$\overline{U_r^2} = \overline{U_r^2}' \left(\frac{\rho'}{1 + \rho'} \right)$$

$$\cong \overline{U_r^{2'}} \left(4 \times 10^{-6} \right)$$

$$= 2.5 \times 10^7 \left(4 \times 10^{-6} \right)$$

$$= 100 \text{ ft}^2$$

$$\cong \overline{\alpha_Q^2}$$

Note that the reduction coefficient,

$$\left(\frac{\rho}{1 + \rho} \right)$$

is always ≤ 1 which guarantees that the measurement process never results in a greater $\overline{U_Q^2}$ after a mark than before it. Suppose in the interval between marks $\overline{U_Q^2}$ does not change much. Then given $\overline{U_Q^2}\ (t_o)$ we have

$$\rho(t_o) = \frac{\overline{\alpha_Q^2}}{\overline{U_Q^2}(t_o)} = \frac{\overline{\alpha_Q^2}}{\overline{U_Q^2}\ (t_o)\left(\frac{\rho'}{1+\rho'}\right)} = \frac{\rho'}{\rho'/(1+\rho')} = 1 + \rho'$$

where $\rho'(t_o) \quad \dfrac{\overline{\alpha_Q^2}}{\overline{U_Q^{2'}}\ (t_o)}$

If, by assumption, ρ changes little between marks, after n marks

$$\rho(t_i) \simeq \rho(t_o) = 1 + \rho'(t_o)$$

$$\rho(t_i) \simeq 1 + \rho'(t_i) = 1 + 1 + \rho' = 2 + \rho'(t_o)$$

$$\rho(t_n) \simeq (n + 1) + \rho'(t_o)$$

thus $$\frac{\rho(t_n)}{1 + \rho(t_o)} = \frac{(n+1) + \rho'(t_o)}{1 + n+1 + \rho'(t_o)}$$

$$\simeq \frac{n + 1}{1 + n + 1} \qquad n + 1 \gg \rho'$$

$$\simeq 1 \qquad n + 1 \gg 1$$

And

$$\overline{U_Q^2}\,(t_n) \simeq \overline{U_Q^{2'}}\,(t_n)$$

I.e. the uncertainty in the direction of $\underline{b}_Q$ is not much reduced after a large number of marks. When this condition has occured, the filter is said to be "saturated". Since E is a real symmetric matrix, it eigenvectors are orthogonal, and the matrix of its normalized eigenvectors is an orthonormal matrix. Hence, as noted in Sec. I, E has a diagonal representation in terms of its eigenvalues as

$$E = P \Lambda P^T \qquad P\,P^T = P^T P = I$$

The matrix of orthonormal vectors, ρ, may be considered as defining a co-ordinate rotation in the space of n dimensions; or alternatively, a change of variable in that space, to a new set

$$\underline{\xi} = P^T \delta X$$

Cast $\underline{w}$ into the new variables

$$\underline{w}_\xi = P^T \underline{w}_{\underline{X}} = \frac{P^T E \underline{b}}{\overline{\alpha_Q^2} + \overline{U_Q^2}}$$

$$= \frac{P^T E \; P P^T \underline{b}}{\overline{\alpha_Q^2} + \overline{U_Q^2}}$$

$$= \frac{\Lambda \; \underline{b}_\xi}{\overline{\alpha_Q^2} + \overline{U_Q^2}} \qquad \underline{b}_\xi = P^T \underline{\xi}$$

Note that

$$\underline{b}_\xi = P^T \frac{\partial Q}{\partial \underline{X}} = \frac{\partial Q}{\partial \underline{\xi}}$$

Also

$$\overline{U_Q^2} = \underline{b}^T E \underline{b} = \underline{b}_\xi^T \Lambda \; \underline{b}_\xi^T = \sum \left(\frac{\partial Q}{\partial \underline{\xi}}\right)_i^2 \sigma_i^2 =$$

$$(\underline{w}_{\xi})_i = \frac{\frac{\partial Q}{\partial \xi i}\, \sigma_i^2}{\overline{\alpha_Q^2} + \sum \left(\frac{\partial Q}{\partial \underline{\xi}}\right)_j^2 \sigma_j}$$

$$= \frac{\partial Q}{\partial \xi_i}\left[\frac{\overline{\alpha_Q^2}}{\sigma_i{}^2} + \left(\frac{\partial Q}{\partial \xi_i}\right)^2 + \sum_{j \neq i}\left(\frac{\partial Q}{\partial \xi_j}\right)^2 \left(\frac{\sigma j}{\sigma_i}\right)^2\right]^{-1}$$

$$= \left(\frac{\partial Q}{\partial \xi_i}\right)^{-1} \left\{1 + \left(\frac{\partial Q}{\partial \xi_i}\right)^{-2} \left[\frac{\overline{\alpha_Q^2}}{\sigma_i{}^2} + \sum_{j \neq i}\left(\frac{\partial Q}{\partial \xi_j}\right)^2 \left(\frac{\sigma_j}{\sigma_i}\right)^2\right]\right\}^{-1}$$

$$= \left(\frac{\partial \xi_i}{\partial Q}\right) \left\{1 + \left(\frac{\partial \xi_i}{\partial Q}\right)^2 \left[\frac{\overline{\alpha_Q^2}}{\sigma_i{}^2} + \sum_{j \neq i}\left(\frac{\partial Q}{\partial \xi_j}\right)^2 \frac{\sigma_j{}^2}{\sigma_i{}^2}\right]\right\}^{-1}$$

Where σ_i^2 is the i^{th} eigenvalue of $\underline{E}$, the variance of ξ_i. In the event that $\sigma_i \gg \sigma_j$ for all j and $\sigma_i \gg \alpha_Q^2$ the inverse in brackets approximates 1:

$$(\underline{w}_{\xi}) \simeq \frac{\partial \xi i}{\partial Q}$$

The updating equation in terms of the new variables is

$$\delta\hat{\underline{\xi}} = P^T \delta\hat{\underline{X}} = P^T \delta\hat{\underline{X}}' + P^T \underline{w} (\delta\tilde{Q} - \delta\hat{Q}')$$

$$= \delta\hat{\xi}' + \underline{w} (\delta\tilde{Q} - \delta\hat{Q}')$$

For the case where $\delta\underline{\hat{\xi}}' = \underline{0}$ as described previously, the update for the $i^{\underline{th}}$ component of $\delta\underline{\hat{\xi}}$ is

$$(\delta\underline{\hat{\xi}})_i \simeq \left(\frac{\partial\xi}{\partial Q}\right)_i \delta\tilde{Q}$$

as expected. Thus thus true meaning of $\underline{w}$ is this:

> It's sort of the inverse partial of $\underline{X}$ with respect to Q, weighted by assorted variances of the measurement and state uncertainties, and conditioned by the extent to which $\underline{b}$ is in the direction of the expected error; sort of . . .

Bandito
Rendezvous
Prox

2.9 Rendezvous Analysis

2.9.1 Introduction

Concentric Flight Plan is the name given to the technique of rendezvous developed during Program Gemini and now in use for Apollo. It is the fruition of attempts to construct a plan which offers simplicity of operation, high reliability of achievement and fuel economy. As a matter of basic philosophy, it has been assumed that the spacecraft crew would participate in the rendezvous activities to the extent of flying the vehicle, evaluating the progress of the trajectory and (when necessary) computing backup solutions for the maneuvers.

Early studies by Flight Procedures Branch and others resulted in the identification of transfer elevation angle (η_{TPI}) and transfer interval ($\overline{\omega t}$) as critical parameters characterizing the shape of various intercept trajectories. Further work resulted in choices of $\overline{\omega t}$ and η_{TPI} to reconcile conflicting requirements of fuel economy, error propagation and ease of operation. As a logical extension of this work, the coelliptical trajectory was settled on as the standard pre-transfer condition. Since the time of arrival at a given elevation angle can be varied by changing differential altitude in the coelliptical phase, it is possible to control the transfer time for a given η_{TPI} , placing it so as to satisfy requirements of lighting, navigation, and ground tracking. By fixing η_{TPI} and $\overline{\omega t}$ in advance and choosing a coelliptical pre-transfer condition, the shape of the rendezvous trajectory is held constant throughout from various approaches, so that the crew can hope to become familiar with its development during training. Such familiarity enables them to monitor its progress, detect off-nominal conditions and develop a high degree of proficiency in execution. The predictions of digital analysis and simulation with regard to trajectory behavior and fuel consumption appear to have been borne out in detail on the Gemini and Apollo programs.

A principle activity of Flight Procedures Branch during Projects Gemini and Apollo has been the construction and testing of rendezvous maneuver charts. These are provided as an independent source of rendezvous maneuver solutions, requiring only the most fundamental information - range, range rate, elevation angle - available directly from the radar and platform. In order to perform the required analysis and verification, several large

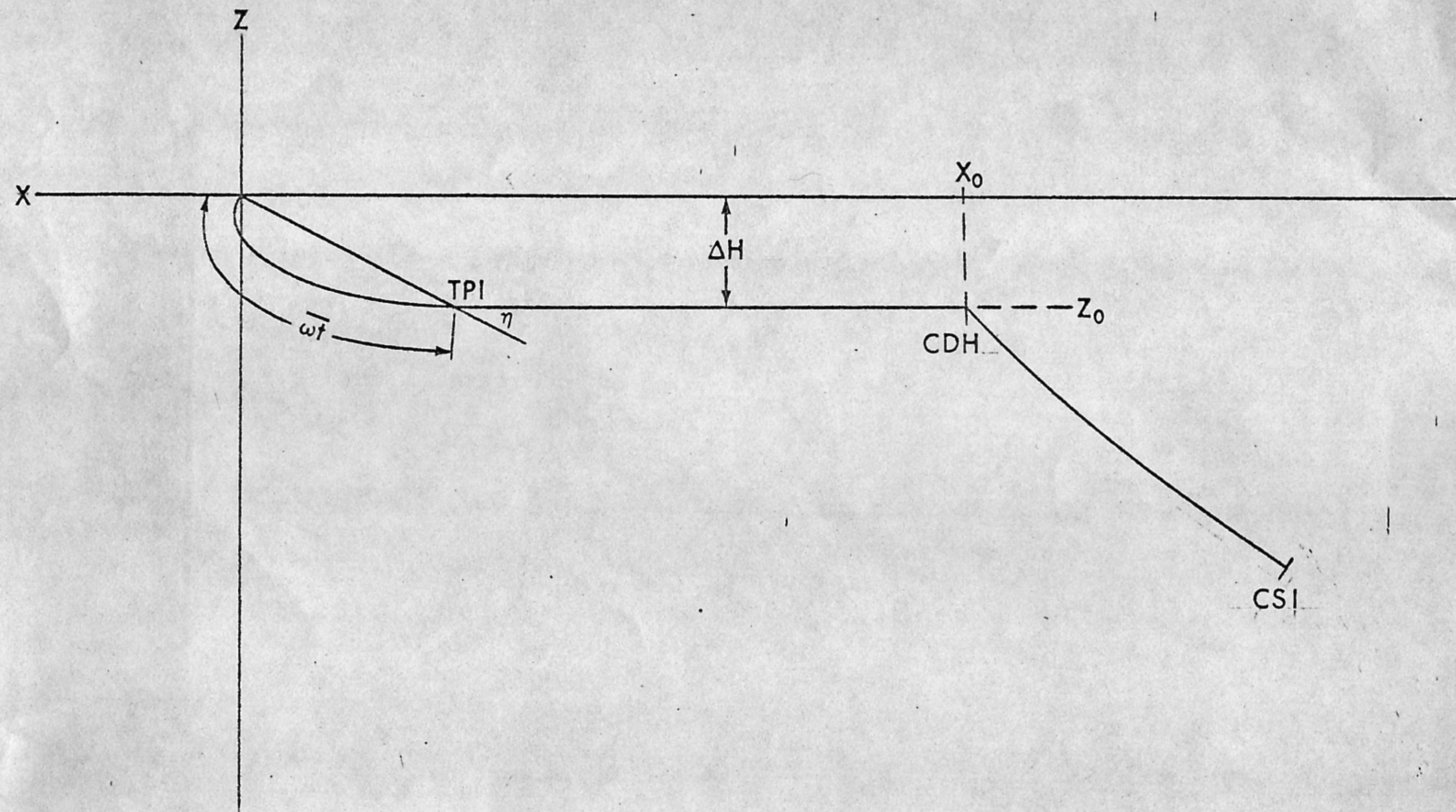


FIGURE 1 — CONCENTRIC FLIGHT PLAN

digital programs capable of solving for various maneuvers and providing dispersed trajectories have been constructed. Among the original techniques developed may be noted the derivation of maneuver functions for TPI and CDH, and the iterative use of the Clohessy-Wiltshire equations to compute transfer maneuver.

2.9.2 Development of Concentric Flight Plan

2.9.2.1 Transfer Phase

In order to clearly display the relative motion of the rendezvous trajectory, solve the linearized equations of relative motion (Eq. 4.1 - 9) for the required relative velocity

$$\dot{\mathcal{S}}_{TPI} = -B^{-1}(\overline{\omega t})\, A\,(\overline{\omega t})\, \mathcal{S}_{TPI} \qquad 2.1 - 1$$

therefore, in the intercept

$$\mathcal{S}(t) = \left[A(\omega t) - B(\omega t)B^{-1}(\overline{\omega t})A(\overline{\omega t})\right]\mathcal{S}_{TPI} \qquad 2.1 - 2$$

These equations have the form

$$\begin{aligned} x(t) &= k_1(t)\, x_{TPI} + k_2(t)\, z_{TPI} \\ z(t) &= h_1(t)\, x_{TPI} + h_2(t)\, z_{TPI} \end{aligned} \qquad 2.1 - 3$$

Since the shape of the trajectory in time is characterized by the ratio of x to z (Figure I),

$$\frac{x(t)}{z(t)} = \cot(\eta) = \frac{k_1 x_{TPI} + k_2 z_{TPI}}{h_1 x_{TPI} + h_2 z_{TPI}} = \frac{k_1 \cot(\eta_{TPI}) + k_2}{h_1 \cot(\eta_{TPI}) + h_2} \qquad 2.1 - 4$$

This equation clearly shows that the shape of an intercept is completely specified by choosing $\overline{\omega t}$ and η_{TPI}. By defining the parameters in advance for all intercepts, the shape is thus fixed, and learning is facilitated.

Practical choices of $\overline{\omega t}$ and η_{TPI} must reconcile fuel economy and ease of control. Actual simulation and flight experience has resulted in a choice of $\overline{\omega t}$ at about 130°. Shorter transfer intervals tend to be more costly in terms of transfer and braking ΔV and longer ones suffer from deleterious propagation of initial errors in estimate of transfer ΔV into miss distances at intercept. It also has proven possible to pick η_{TPI} such that the apparent inertial motion of the target in the latter part of the intercept is near zero and so that the transfer

from a coelliptical orbit is along the line of sight. For lunar orbit, this is about 26.6° and for earth orbit 27.5°. The practical advantage of this choice is that the terminal braking procedure is particularly simple: the pilot thrusts so as to null any apparent inertial motion of the target normal to the line of sight.

2.9.2.2 Coelliptical Phase

It is generally desired to constrain the time of transfer so as to satisfy operational constraints such as lighting and tracking. For given η_{TPI}, this reduces to the problem of bringing about the appearance of this angle at a selected instant. By far the simplest pre-transfer condition (with standard approach conditions) which allows this is the co-elliptical trajectory, wherein the differential altitude is constant throughout the phase. Again, from the equations 4.1 - 8a, for coellipticity

$$\begin{aligned} x(t) &= x_o + \tfrac{3}{2} z_o \omega t \\ z(t) &= z_o = \Delta h \end{aligned} \qquad 2.2 - 1$$

or

$$\cot(\eta) = \cot(\eta_o) + \tfrac{3}{2}\omega t \qquad 2.2 - 2$$

Since at transfer $\eta(t) = \eta_{TPI}$, it is required that

$$\cot \eta_o = \cot \eta_{TPI} - \tfrac{3}{2}\omega t_{TPI}$$

The η_o point at which a maneuver is performed to bring about the co-elliptical condition is customarily designated CDH (Constant Delta-H). It is here implied that some maneuver(s) has been performed prior to CDH so as to bring about the required $\cot(\eta_o) = x_{CDH}/z_{CDH}$. In the Apollo program, this maneuver is called CSI (Concentric Sequence Initiation) and is customarily performed $\frac{1}{2}$ revolution before the CDH point. One should note that at first order, the appropriate value of η_o is a function only of the time from CDH to TPI.

The values of η_{TPI} and $\overline{\omega t}$ now in use represent compromise choices and may be inappropriate under some special conditions. If, for example, it is desired to rendezvous under circumstances where the transfer maneuver is subject to errors, the effect of these may be minimized by shortening $\overline{\omega t}$, as was done for the GT-10 passive rendezvous.

2.9.2.3 Targeting Phase

For a given approximate time of CDH, it is necessary to arrange

x_{CDH}, z_{CDH} such that their ratio ($\cot(\eta_{\overline{CDH}})$) has the correct value. As there are two degrees of freedom and only one condition to be satisfied, the restriction can be made in several ways. If a value of z_{CDH} is given and the time of CSI fixed, the value of x_{CDH} is constrained and can be obtained by one two-axis maneuver at CSI or two single-axis maneuvers at different times. Alternatively, by letting z_{CDH} go unrestrained, the ratio can be obtained by means of a single one-axis maneuver. For reasons of efficiency, this is currently defined to be a horizontal maneuver n/2 periods before CDH. Under this plan, the value of Δh is a variable. A maneuver added to the sequence which has as its objective changing the trajectory so as to bring about a certain value of Δh is called a height adjust (N_H) maneuver.

It may be here noted that the out-of-plane problem can be shown to be uncoupled (to second order) from the in-plane rendezvous problem. Therefore, out-of-plane solutions may be handled separately in computation and application. In practice, as soon as the out-of-plane motion has been established, a maneuver is performed in conjunction with a scheduled in-plane maneuver (such as CSI) which has the effect of nulling the current out-of-plane rate, thus forcing a node to occur $\frac{1}{4}$ rev later. On arrival at this node, the velocity is again nulled, placing the active vehicle in-plane. This sequence may be repeatedly performed as better information is obtained until a satisfactory condition exists. Small residual errors in the out-of-plane direction are easily handled during terminal braking.

2.9.2.4 Operational Constraints

Considerations which determine the timing and arrangement of maneuvers are ground tracking, spacecraft-target visibility and maintenance of the trajectory plan. The most critical of these requirements constrain the time of transfer, which must occur so that the target is visible during tracking, and so that suitable tracking periods are available to each vehicle. A detailed analysis of the requirements is contained in MSC Internal Note CF-R-69-6. In order to maintain the accuracy of the TPI maneuver and minimize the effects

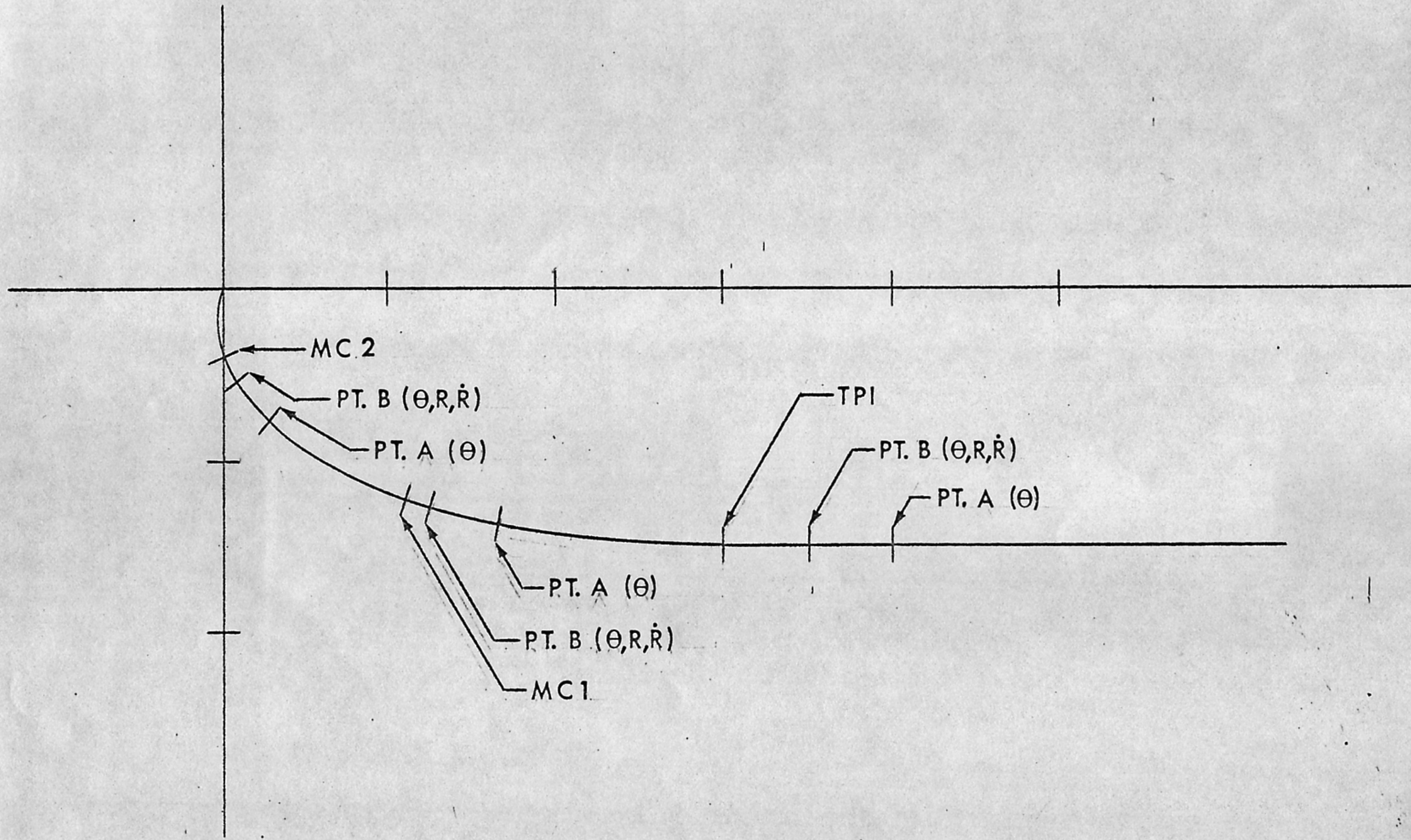


FIGURE 11

of trajectory estimation errors, it is generally undesirable for Δh to be smaller than some value. If an analysis of the likely trajectory dispersions indicates this may happen, or that it may grow unacceptably large, an N_H maneuver may be inserted into the flight plan to minimize these effects.

2.9.3 Backup Charts

2.9.3.1 TPI Backups

The method employed in computing a backup TPI solution is that of differencing the actual conditions observed just prior to TPI from the conditions required at TPI to achieve rendezvous $\overline{\omega t}$ of orbit travel later. The observables required for this technique are range (R) and range rate ($\dot{R}$) at a fixed time before TPI and two measurements of relative elevation angle (θ) at fixed times before TPI. The solution obtained is resolved into a velocity component along the line-of-sight (ΔV_{LOS}) at TPI and a velocity component normal to the line-of-sight (ΔV_{NOR}) at TPI. Figure II shows the measurement geometry.

The algorithms used to compute the velocity along the line-of-sight and the velocity normal to the line-of-sight are

$$\Delta V_{LOS} = \left[\frac{\dot{R}_{B(N)} + \Delta V_{LOS(N)}}{R_{B(N)}}\right] R_B - \dot{R}_B$$

$$\Delta V_{NOR} = \left[\frac{\Delta V_{NOR(N)}}{R_{B(N)}} - \frac{\Delta\theta_{(N)} - \Delta\theta}{\Delta t}\right] R_B \qquad 3.1 - 1$$

where

$$\Delta\theta_{(N)} = \theta_{B(N)} - \theta_{A(N)}$$

$$\Delta\theta = \theta_B - \theta_A$$

and Δt is the time between points A and B. The subscripts A and B indicate the time at which an observable was measured, A being at a time earlier than B. The subscripts N indicate those quantities which are referenced to the nominal trajectory.

This approach depends on the fact that the catchup rate ($\dot{X}$) of the active vehicle with respect to the passive vehicle is very nearly constant for coelliptic orbits, and is given by

$$\dot{x} = \tfrac{3}{2}\omega z_0$$

where ω is orbital angular rate and $\bar{z}_o$ is the differential altitude between the two vehicles (see the derivation in section 2.9.4).
It can be shown in the following manner that range rate is a function of catchup rate and relative elevation angle.
Letting

$$R = \sqrt{x^2 + z^2}$$

and differentiating, we get

$$\dot{R} = [\dot{x}x + \dot{z}z] / R \qquad 3.1 - 2$$

But, under the assumption that we are coelliptic, $\dot{z} = 0$ and $x/R \simeq \cos\theta$ so

$$\dot{R} = \dot{x}\cos\theta \qquad 3.1 - 3$$

Similarly, one can show that $\dot{\theta}$ is a function of elevation angle:

$$\theta = \tan^{-1}[z/x]$$

$$\dot{\theta} = [x\dot{z} - z\dot{x}] / R^2 \qquad 3.1 - 4$$

But $\dot{z}=0$, $\dot{x}=3/2\,\omega \bar{z}_o$, $z=z_o$ so

$$\dot{\theta} = -\frac{3}{2}\omega\left(\frac{z_o}{R}\right)^2 = -\frac{3}{2}\omega\sin^2(\theta) \qquad 3.1 - 5$$

Hence, the values of θ, $\dot{R}$, and θ at B can be used to infer $\dot{R}$ and $\dot{\theta}$ at TPI. These predicted values can thus be differenced from the required values to get a TPI solution.

The backup chart is graphical in nature and the data used to plot it are computed using a digital routine which generates the orbital parameters for a set of trajectories covering the region of expected dispersions about the nominal trajectory. The outputs of the routine are $(\dot{R}/R)_{REQ}$ and θ_{REQ} as functions of θ_B (the subscript $_{REQ}$ denoting the required values of the variables). For each of the θ_B's under consideration, coelliptic orbits are generated and advanced back in time by the appropriate Δt to get θ_A and hence, $\Delta\theta$. The trajectories are then advanced to TPI and the transfer maneuver is computed (see section 2.9.4.3), ΔV_{LOS} and ΔV_{NOR}. Then

$$\dot{R}/R = [\dot{R}_B + \Delta V_{LOS}] / R_B \qquad 3.1 - 6a$$

and

$$\Delta\theta_{REQ} = \Delta\theta - \left[\frac{\Delta V_{NOR}}{R_B}\right]\Delta t \qquad 3.1 - 6b$$

are computed and stored.

G MISSION TERMINAL PHASE INITIATION

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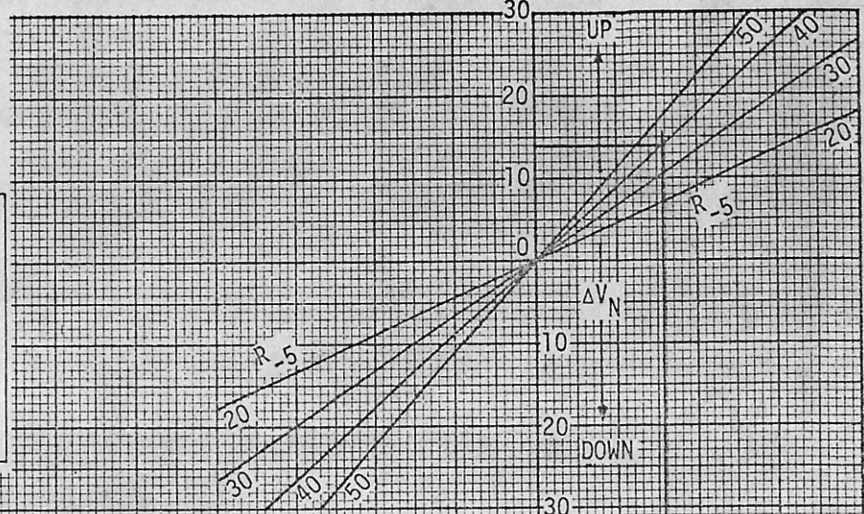
$\dot{R}_{REQ}$ 150.6 136.9

θ_{-5} 23.50 22.24 R_{-5} 40.00 38.18 $\dot{R}_{-5}$ -120.3 112.3

θ_{-9} -20.00 19.54 $\Delta\dot{R}$ 30.3 24.6

$\Delta\theta$ 3.50 2.70

	PNGS (N59)	GND	CHARTS	ΔT
$\pm$F/A			30.3 F	
$\pm$R/L				
$\pm$D/U			14.0 D	



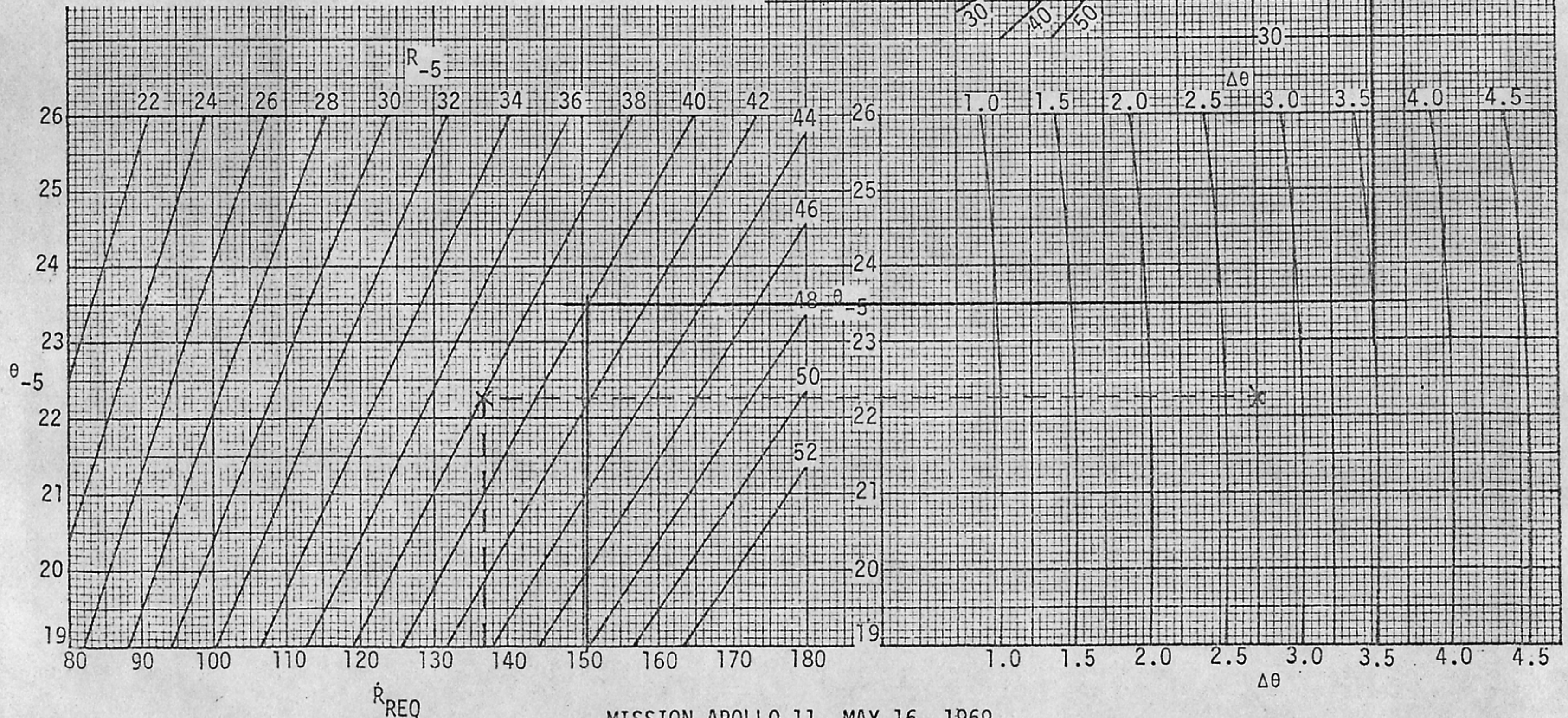


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FIGURE III

Graphically, the solution for ΔV_{LOS} is obtained by taking the product of $(\dot{R}/R)_{REQ}$ and R_B to get $\dot{R}_{REQ}$. $\dot{R}_B$ is then subtracted from $\dot{R}_{REQ}$ to get ΔV_{LOS}. Likewise, $\Delta\theta_{REQ}$ is subtracted from the actual $\Delta\theta$ and multiplied by $R_B/\Delta t$ to get ΔV_{NOR}. Figure III presents the TPI chart as flown on Apollo 11.

It should be noted here that the midcourse correction (MCC) charts are equivalent to TPI charts with the following exceptions:

1. The active vehicle is assumed to be on a collision course with respect to the passive vehicle rather than in a coelliptic orbit as in the nominal case.

2. The MCC burn is assumed to occur at the instant of the second measurement point, rather than a fixed time later.

With the exception of the above assumptions, the MCC charts are generated and used in exactly the same manner as the TPI chart. Later work on this problem has resulted in a TPI table, utilizing R, $\dot{R}$, θ, and the interval (Δt) between the last measurement and TPI.

2.9.3.2 CSI Backups

Since the ΔV for CSI is not, (in general) even approximately available as a function of the observables, it is necessary to adopt a somewhat different approach than that of the last section. Of the standard mathematical techniques for approximating an unknown function; the simplest is the Taylor Series. For CSI, only the in-plane problem is to be solved; therefore, four independent measurements will serve to constrain the problem. The simplest possibility is an equation of the form

$$\Delta V_{CSI} = \sum_n a_n q_1^n + b_n q_2^n + c_n q_3^n + d_n q_4^n \qquad 3.2 - 1$$

i.e., an uncoupled power series in the observables q_i. Should this assumption fail, it may be necessary to look for higher order cross terms in particular cases. For the situation where CSI occurs as the result of a nominal ascent from the lunar surface, and similar trajectories, this assumption works well.

To determine the constants a_n, b_n, c_n, d_n, a set of trajectories

spanning the envelope of expected dispersions is required, together with the ΔV_{CSI} for each case. In the solution of this problem, measurement time is held fixed to remove it as a variable, and the values of the observables, R and/or $\dot{R}$ are read at each measurement point. When these are substituted into 3.2 - 1, a system of simultaneous algebraic equations result

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \Delta V_K \end{bmatrix} = \begin{bmatrix} q_1 & q_1^2 & \cdot & \cdot & q_2 & q_2^2 & \cdot & \cdot & q_3 & q_3^2 & \cdot & \cdot & q_4 & q_4^2 & \cdot & \cdot \\ \cdot & & & & & & & & & & & & & & & \\ \cdot & & & & & & & & & & & & & & & \\ \cdot & & & & & & & & & & & & & & & \\ \cdot & & & & & & & & & & & & & & & \\ \cdot & & & & & & & & & & & & & & & \\ \cdot & & & & & & & & & & & & & & & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ d_1 \\ d_2 \\ \cdot \end{bmatrix}$$

This system is generally well over determined, and writing

$$\overrightarrow{\Delta V} = Q \vec{C}$$

by the usual theory (2.9.4.6)

$$\vec{C} = [Q^t Q]^{-1} Q^t \, \widehat{\Delta V}$$

is the least squares fit of $\vec{C}$ to the system. Having determined these constants, a table of partial sums is constructed where each entry is the partial sum of a particular value of a q_i for various values. Such a table is presented in Figure IV. For this case, the measurements are $\dot{R}$ at CSI -30^m, -20^m, -10^m and a range at -10^m. As each measurement is obtained, its corresponding factor is looked up in the table and logged in the space provided. At the last observation, these factors are summed to give the ΔV_{CSI} .

2.9.3.3 CDH Backups

Early work on the CDH problem by Flight Procedures Branch elicited the property that range rate in a near-coelliptical orbit is closely representable by a sinusoid of the form

$$\dot{R} = \dot{R}_o + \dot{R}_m \cos(\phi + \Delta\omega t) \qquad 3.3 - 1$$

By appeal to the linearized relative equation it can also be shown that

D MISSION CDH CHART

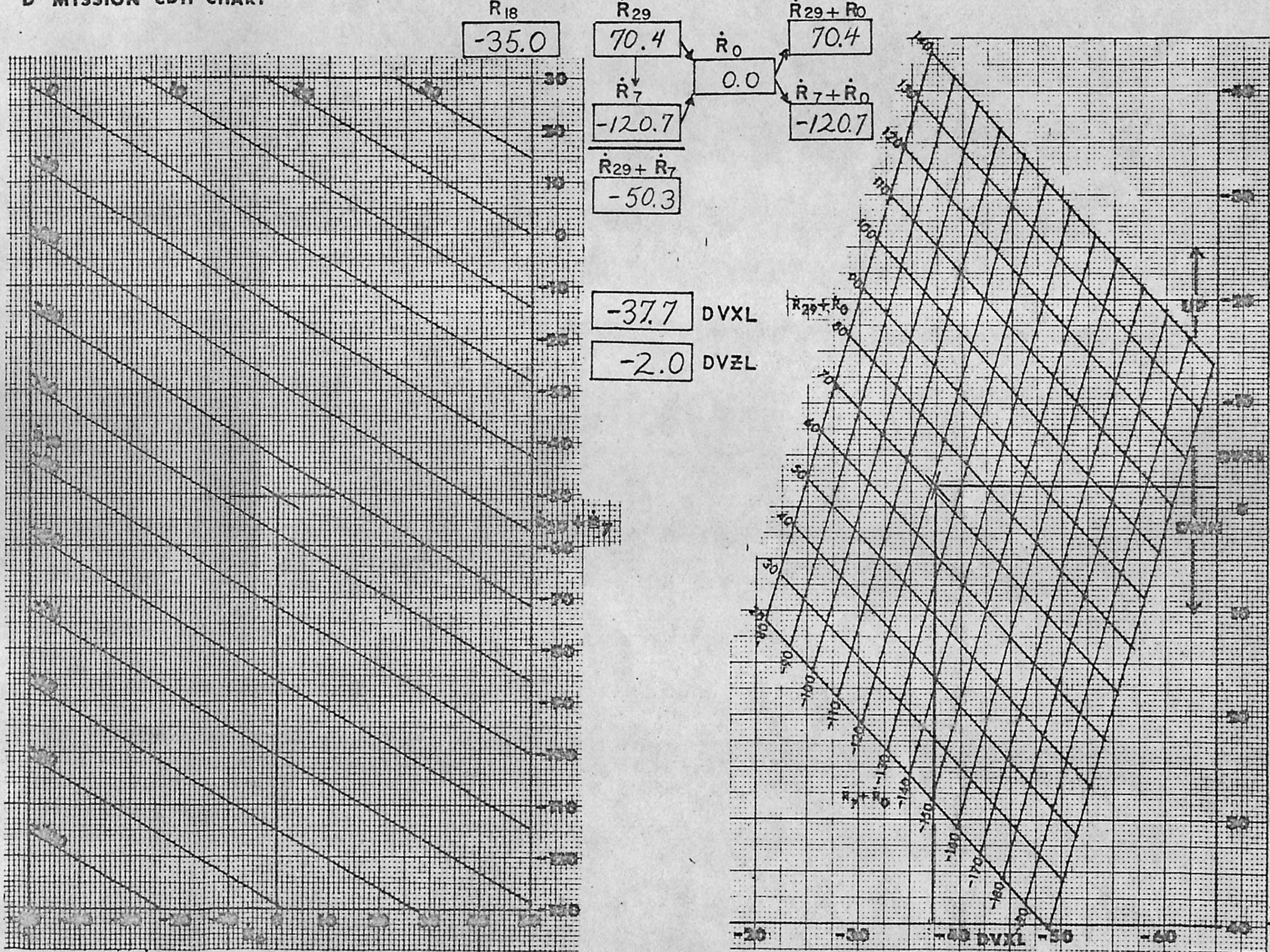

FIGURE V

$\dot{R}1$	F1	$\dot{R}2$	F2	$\dot{R}3$	F3	R3	F4
-240.0	247.3	-140.0	254.7	-70.0	72.4	120.0	15.2
-241.0	248.4	-141.0	256.6	-71.0	73.4	121.0	15.5
-242.0	249.4	-142.0	258.4	-72.0	74.4	122.0	15.7
-243.0	250.5	-143.0	260.2	-73.0	75.5	123.0	15.9
-244.0	251.5	-144.0	262.1	-74.0	76.5	124.0	16.1
-245.0	252.5	-145.0	263.9	-75.0	77.5	125.0	16.3
-246.0	253.6	-146.0	265.7	-76.0	78.6	126.0	16.5
-247.0	254.6	-147.0	267.6	-77.0	79.6	127.0	16.7
-248.0	255.7	-148.0	269.4	-78.0	80.6	128.0	16.9
-249.0	256.7	-149.0	271.3	-79.0	81.7	129.0	17.1
-250.0	257.8	-150.0	273.1	-80.0	82.7	130.0	17.3
-251.0	258.8	-151.0	274.9	-81.0	83.7	131.0	17.5
-252.0	259.9	-152.0	276.8	-82.0	84.8	132.0	17.7
-253.0	260.9	-153.0	278.6	-83.0	85.8	133.0	17.9
-254.0	262.0	-154.0	280.4	-84.0	86.9	134.0	18.1
-255.0	263.0	-155.0	282.3	-85.0	87.9	135.0	18.4
-256.0	264.1	-156.0	284.1	-86.0	88.9	136.0	18.6
-257.0	265.1	-157.0	286.0	-87.0	90.0	137.0	18.8
-258.0	266.2	-158.0	287.8	-88.0	91.0	138.0	19.0
-259.0	267.2	-159.0	289.7	-89.0	92.0	139.0	19.2
-260.0	268.3	-160.0	291.5	-90.0	93.1	140.0	19.4
-261.0	269.3	-161.0	293.4	-91.0	94.1	141.0	19.6
-262.0	270.4	-162.0	295.2	-92.0	95.2	142.0	19.8
-263.0	271.4	-163.0	297.0	-93.0	96.2	143.0	20.0
-264.0	272.5	-164.0	298.9	-94.0	97.2	144.0	20.2
-265.0	273.5	-165.0	300.7	-95.0	98.3	145.0	20.5
-266.0	274.6	-166.0	302.6	-96.0	99.3	146.0	20.7
-267.0	275.7	-167.0	304.4	-97.0	100.4	147.0	20.9
-268.0	276.7	-168.0	306.3	-98.0	101.4	148.0	21.1
-269.0	277.8	-169.0	308.1	-99.0	102.4	149.0	21.3
-270.0	278.8	-170.0	310.0	-100.0	103.5	150.0	21.5
-271.0	279.9	-171.0	311.9	-101.0	104.5	151.0	21.7
-272.0	281.0	-172.0	313.7	-102.0	105.6	152.0	21.9
-273.0	282.0	-173.0	315.6	-103.0	106.6	153.0	22.1
-274.0	283.1	-174.0	317.4	-104.0	107.7	154.0	22.4
-275.0	284.2	-175.0	319.3	-105.0	108.7	155.0	22.6
-276.0	285.2	-176.0	321.1	-106.0	109.7	156.0	22.8
-277.0	286.3	-177.0	323.0	-107.0	110.8	157.0	23.0
-278.0	287.4	-178.0	324.9	-108.0	111.8	158.0	23.2
-279.0	288.4	-179.0	326.7	-109.0	112.9	159.0	23.4
-280.0	289.5	-180.0	328.6	-110.0	113.9	160.0	23.6
-281.0	290.6	-181.0	330.4	-111.0	115.0	161.0	23.8
q_1	$\Sigma a_n q_1^n$	q_2	$\Sigma b_n q_2^n$	q_3	$\Sigma c_n q_3^n$	q_4	$\Sigma d_n q_4^n$

CSI BACKUP TABLE
MISSION G

TIME (Min)			NOMINAL
-30	$\dot{R}1$	-278.0	(-283.3)
-20	$\dot{R}2$	-170.0	(-173.9)
-10	$\dot{R}3$	-91.0	(- 94.0)
-10	R3	149.0	(154.1)
	F1	287.4	(293.0)
	+F3	94.1	(97.2)
		381.5	(390.2)
	-F2	310.0	(-317.3)
		71.5	(72.9)
	-F4	21.3	(- 22.4)
		50.2	(50.5)
	+ΔΔVCSI	0.0	(0.0)
	ΔVCSI	$\boxed{50.2}$	(50.5)

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FIGURE IV

$$\Delta V_x = -\frac{1}{A}\dot{R}\cos(\bar{\psi}) \qquad \bar{\psi} = \phi + 2\Delta\omega t + \alpha \qquad 3.3 - 2$$

$$\Delta V_z = \frac{2}{A}\dot{R}\sin(\bar{\psi})$$

where $A \simeq 4$ and $\Delta\omega t$ is the interval between measurements, α the interval between the last measurement and CDH. Since the CDH maneuver depends only on the relative velocity and current Δh, 3 independent measurements suffice to solve the problem as reflected by 3.3 - 1. From the equations 3.3 - 1 and 3.3 - 2, a nomographic solution of the CDH problem may be constructed as presented in Figure V. The somewhat lengthy derivation of the actual results is presented in Section 2.9.4.2.

Later analysis directed at the solution of the CSI problem has resulted in the techniques of the last section being applied to the construction of CDH backup tables. Their preparation and use is exactly the same as for CSI.

2.9.3.4 Performance Analysis

Once backup charts have been generated, a statistical analysis is done to determine exactly how well they can be expected to perform. Data for the analysis are generated with a routine which executes a large number of rendezvous, and calculates statistical data on the parameters of interest.

The runs for the analysis generally start approximately 40 minutes prior to CSI, and are run through intercept. A total of 300 runs are generally made, broken up into four groups, each run having randomly dispersed initial conditions. The first of these groups consists of 100 runs, made with all applicable random errors, biases, and drifts. It had previously been determined that 100 runs would yield statistically meaningful results. This was done by plotting some of the randomly distributed variables and noting the shape of the resulting bell curve. Studies of this type also indicated that 50 runs would be the absolute minimum number that could be made, and still yield meaningful results.

A second group also consists of 100 runs. These runs are identical to the first set, except for the fact that braking and line-of-

TABLE 1

Differences Between Chart Solutions With and Without Errors and Conic Solutions

Maneuver	Average Set D ft/sec	Average Set A ft/sec	Mean Set D ft/sec	Mean Set A ft/sec	Standard Deviation Set D ft/sec	Standard Deviation Set A ft/sec
*CSI $\Delta\Delta V_H$	0.0	.77	0.0	-.02	0.0	.95
CDH $\Delta\Delta V_V$	1.11	1.54	1.11	1.05	1.21	1.83
CDH $\Delta\Delta V_H$	.25	.46	-.25	-.33	.34	.60
TPI $\Delta\Delta V_{LOS}$	1.34	2.40	-1.04	-.76	1.45	3.02
TPI $\Delta\Delta V_N$	.37	2.41	-.29	-.44	.54	3.12
MCC1 $\Delta\Delta V_{LOS}$	1.59	2.65	-1.59	-1.83	.91	2.80
MCC1 $\Delta\Delta V_N$	1.34	2.16	1.34	1.47	.37	2.26
MCC2 $\Delta\Delta V_{LOS}$	.84	1.48	-.82	-.97	.59	1.48
MCC2 $\Delta\Delta V_N$	.50	1.29	.50	.85	.16	1.77

The data in Table 3-3 listed under Set D represents the theoretical error inherent in the charts, while the data listed under Set A represents the total expected error, including theoretical error, system errors, and execution errors.

*The value for CSI $\Delta\Delta V$ represents the difference between CSI ΔV computed with sensor and reading errors, and the value for CSI ΔV computed without sensor and reading errors.

sight control are omitted. This is done in order to obtain data on miss distance at closest approach.

The remaining two sets consist of 50 runs each, and are made without any random errors, biases, or drifts. The purpose of these two sets was to get baseline data on chart performance. Again, the second set was identical to the first, with the exception that braking and line-of-sight control were omitted to establish data on miss distance.

During the analysis, one of the basic parameters which was looked at, was the accuracy of the chart solutions. Table 1[1] is representative of the type of data which were derived. In addition, data concerning miss distance at closest approach, total fuel used in the rendezvous, and arrival time at TPI were also derived.

2.9.3.5 Onboard Rendezvous Evaluation

Evaluation of the progress of the rendezvous is one of the primary crew functions in manual spaceflight. Much of the analysis done by Flight Procedures Branch has been directed toward the provision of "rule of thumb" statements about the behavior of maneuver solutions following trajectory dispersions. For this purpose, the linearized equations of Section 2.9.4 constitute a powerful analytical tool. Let the coordinate system of Section 2.9.4 be fixed on the nominal spacecraft so that motion of the actual to nominal may be compared. As an example, consider an insertion dispersed behind the nominal phase angle (up range distance). To first order, the equations say that this dispersion will propagate to a similar off-nominal position at each point, in particular at CDH. If the phase angle was too large, the S/C is too far up range and a larger Δh will be necessary to arrive at η_{TPI} at the right time. For a nominal rendezvous profile, this will result in a lower ΔV_{CSI}, since CSI is raising pericythion from about 10 nm to about 45 nm. Similar remarks apply to a horizontal overspeed at insertion. The equations show that after nearly 1 rev, i.e., at CDH, an overspeed will place the spacecraft up range of its

1. MSC Internal Note No. CF-R-69-20, Apollo Mission F Performance Analysis of Rendezvous Charts, April 29, 1969.

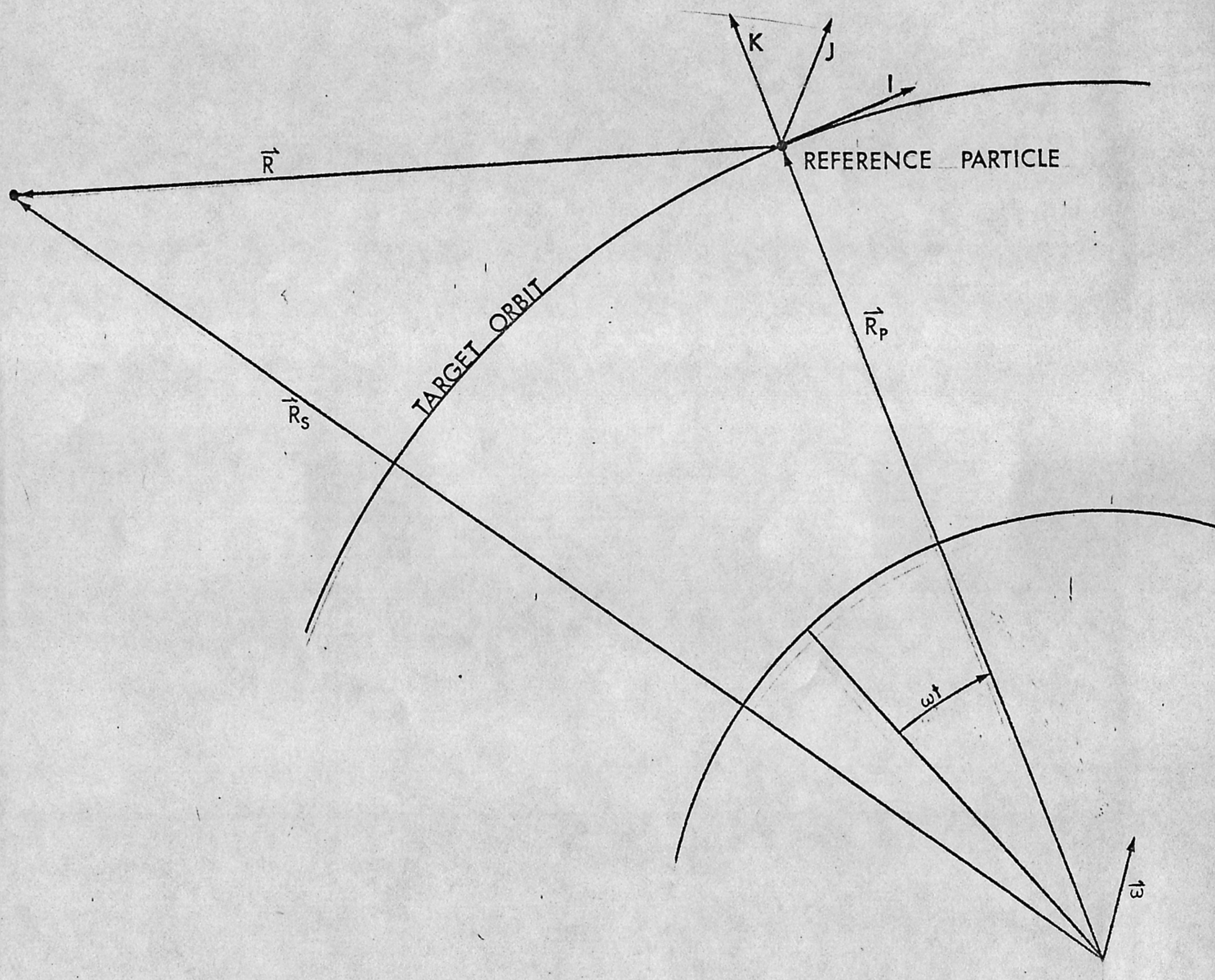


FIGURE VI — TWO PARTICLES IN ORBIT AROUND A SPHERICAL PLANET

nominal position. Converse statements apply to too small an insertion phase angle or an insertion underspeed. If the insertion error is in altitude rate, the actual spacecraft after one rev is nearly coincidental with the nominal one, but has an altitude rate error nearly equal to that at insertion. Thus the vertical component of the CDH maneuver will be perturbed to remove it, but the resulting Δh will be little affected. Provided the pilot has information on the dispersions resulting from a particular case, he can infer the trend of his maneuver solutions in comparison to nominal.

By similar means, the effect of an incorrect CSI maneuver in arrival time at η_{TPI} may be gauged. Since CSI is strictly horizontal, one need only consider dispersions in this axis and it is instantly apparent that an overburn causes late arrival and conversely. For lunar orbit, this is about 4 min/fps.

2.9.4 Mathematical and Technical Appendix

2.9.4.1 The Linearized Relative Equations

All rendezvous problems have in common two basic requirements; (1) knowledge of the relative motion between the interceptor and target, and (2) a plan of maneuvers for the interceptor which results in a terminal condition of zero relative velocity at a small distance.

In order to facilitate analysis and understanding of the rendezvous problem, it is useful to develop a set of equations describing the relative motion of one vehicle with respect to another when they are reasonably close. This follows the standard treatment, and two assumptions will be adherred to in the discussion:

1. The orbit of the reference particle is near circular.
2. The distance between them is small compared to the radius vector of the reference particle.

In Fig. IV, construct a coordinate system fixed on the reference particle: $\hat{K} = \vec{R}_P / |\vec{R}_P|$, $\hat{J} = \hat{K} \times \dot{\vec{V}}_P / |\hat{K} \times \vec{V}_P|$, $\hat{I} = \hat{J} \times \hat{K}$ and defining

$$\mathbb{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \Omega = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

and noting Ω and R_p constant by assumption 1., the derivative operator for such a vector is

$$\frac{d\vec{A}}{dt} = [I\frac{\partial}{\partial t} + \Omega]\vec{A}$$

$$\frac{d^2\vec{A}}{dt^2} = [I\frac{\partial^2}{\partial t^2} + 2\Omega\frac{\partial}{\partial t} + \Omega^2]\vec{A}$$

Using these equations and writing R_s with respect to R_p find Newton's Law for particle S

$$\ddot{\vec{R}}_S = \frac{\partial^2\vec{R}_S}{\partial t^2} + 2\Omega\frac{\partial\vec{R}_S}{\partial t} + \Omega^2\vec{R}_S = -\frac{\mu}{R_S{}^3}\vec{R}_S \qquad \vec{R}_S = \begin{bmatrix} x \\ y \\ z+R_P \end{bmatrix} \qquad 4.1-2$$

$$= \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + 2\omega\begin{bmatrix} \dot{z} \\ 0 \\ -\dot{x} \end{bmatrix} - \omega^2\begin{bmatrix} x \\ 0 \\ z+R_P \end{bmatrix} = -\frac{\mu}{R_S^3}R_S = -\frac{\mu}{R_P^3}\left(\frac{R_P}{R_S}\right)^3\vec{R}_S \qquad 4.1-3$$

Now examine

$$\left(\frac{R_S}{R_P}\right)^{-3} = \frac{\left[x^2+y^2+(z+R_P)^2\right]^{-\frac{3}{2}}}{R_P^{-3}} = \left[\left(\frac{x}{R_P}\right)^2 + \left(\frac{y}{R_P}\right)^2 + \left(1+\frac{z}{R_P}\right)^2\right]^{-\frac{3}{2}} \cong 1 - \frac{3z}{R_P} \qquad 4.1-4$$

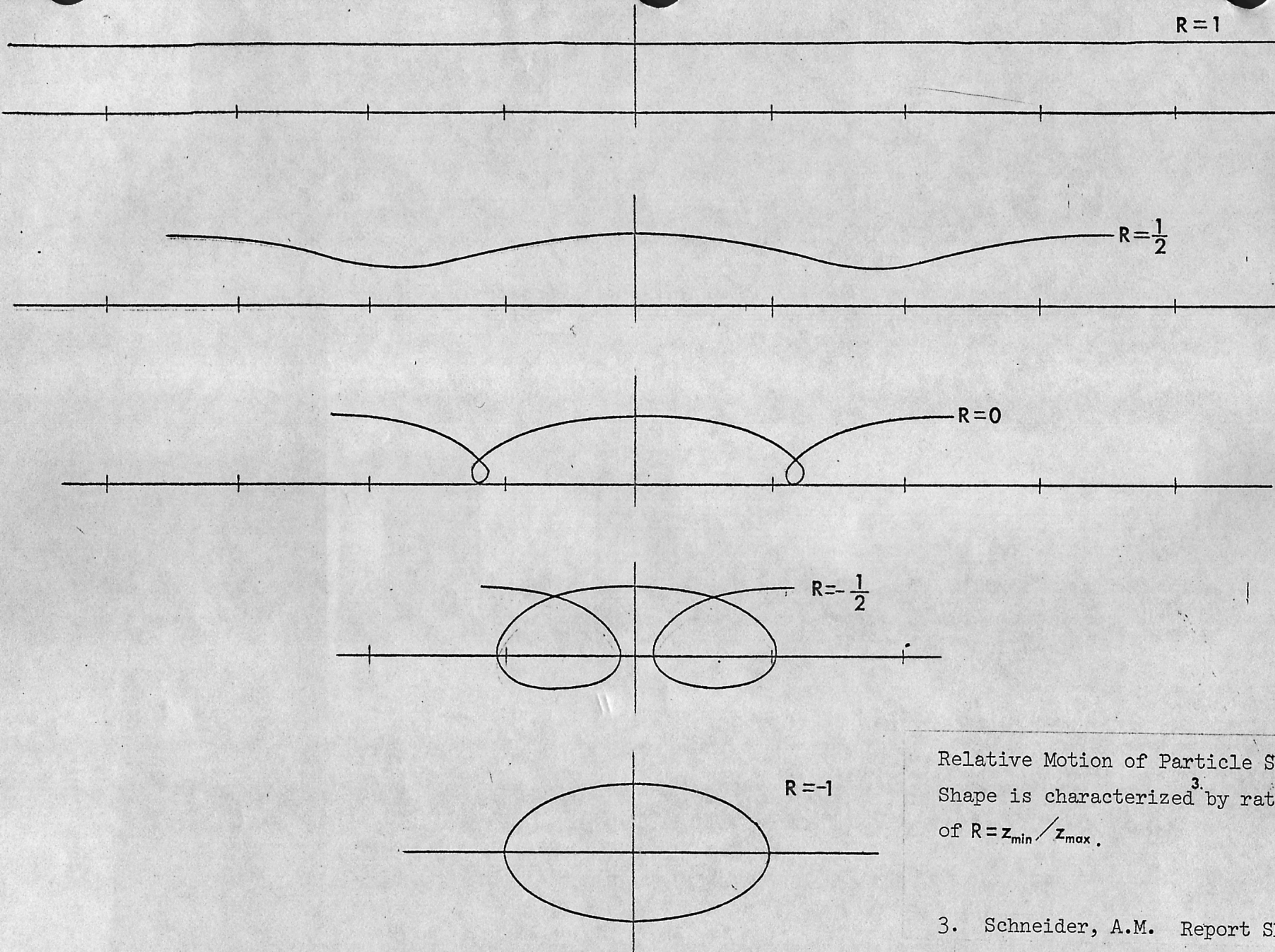
if terms of second order are ignored by assumption 2. This is the first of two approximations to be made in the interest of obtaining a linear system. Careful note should be taken of the implied limitations on the result. The second approximation is gotten by assuming terms of the form

$$\frac{3xz}{R_P}, \quad \frac{3yz}{R_P}, \quad \frac{3z^2}{R_P}$$

are also negligible, thus to get

$$\left(\frac{R_P}{R_S}\right)^3\vec{R}_S \simeq \begin{bmatrix} x \\ y \\ R_P-3z \end{bmatrix} \qquad 4.1-5$$

further identify $-\frac{\mu}{R_P^3} = -\omega^2$ and write



Relative Motion of Particle S;
Shape is characterized[3] by ratio
of $R = z_{min} / z_{max}$.

3. Schneider, A.M. Report SDC-1-69

FIGURE VII

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} + 2\omega\begin{bmatrix}\dot{x}\\ 0\\ -\dot{x}\end{bmatrix} + \omega^2\begin{bmatrix}\cancel{0}\ -x\\ \cancel{y}\ 0\\ -(z+R_P)\end{bmatrix} = -\omega^2\begin{bmatrix}x\\ y\\ (z+R_P)-3z\end{bmatrix} \qquad 4.1 - 6$$

and finally get the system of equations

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} + 2\omega\begin{bmatrix}\dot{z}\\ 0\\ -\dot{x}\end{bmatrix} + \omega^2\begin{bmatrix}0\\ y\\ -3z\end{bmatrix} = 0 \qquad 4.1 - 7$$

These simultaneous equations are readily integrated to give

$$\begin{bmatrix}x\\ z\end{bmatrix} = \begin{bmatrix}1 & -6(\omega t - \sin\omega t)\\ 0 & (4-3\cos\omega t)\end{bmatrix}\begin{bmatrix}x_o\\ z_o\end{bmatrix} + \begin{bmatrix}\frac{4}{\omega}(\sin\omega t - \frac{3}{4}\omega t) & -\frac{2}{\omega}(1-\cos\omega t)\\ \frac{2}{\omega}(1-\cos\omega t) & \frac{\sin\omega t}{\omega}\end{bmatrix}\begin{bmatrix}\dot{x}_o\\ \dot{z}_o\end{bmatrix} \qquad 4.1 - 8a$$

$$y = y_o\cos\omega t + \frac{\dot{y}_o}{\omega}\sin\omega t \qquad 4.1 - 8b$$

Of immediate interest is the fact that the out-of-plane motions are uncoupled from the in-plane. Thus the out-of-plane problem may be treated separately. Writing equations 4.1 - 8a in the form

$$\mathcal{S} = A[\omega t]\mathcal{S}_o + B[\omega t]\dot{\mathcal{S}}_o \qquad 4.1 - 9$$

and considering two possible initial states for the particle

$$\mathcal{S}^1_o, \dot{\mathcal{S}}^1_o \quad \text{and} \quad \mathcal{S}^2_o; \dot{\mathcal{S}}^2_o$$

it can be shown that because of the linearity of the equations, the state at any time due for a combination of initial perturbations is the same as the sum of the states at that time due to the perturbations applied separately, i.e.:

$$\begin{aligned}\mathcal{S} &= A\left[\mathcal{S}^1_o + \mathcal{S}^2_o\right] + B\left[\dot{\mathcal{S}}^1_o + \dot{\mathcal{S}}^2_o\right]\\ &= \left[A\,\mathcal{S}^1_o + B\dot{\mathcal{S}}^1_o\right] + \left[A\,\mathcal{S}^2_o + B\,\dot{\mathcal{S}}^2_o\right]\\ &= \mathcal{S}^1 + \mathcal{S}^2\end{aligned}$$

It may be guessed from intuition and stated from experience that the requirement that the reference particle orbit be circular may be relaxed somewhat, provided that x is interpreted as down-range curvilinear distance along the orbit arc, and z as normal distance from the point x to the particle R_s. It should be stressed that even in regions where the assumptions leading to linearity are not strictly true, the equations still provide a useful indication of the relative motion to be expected.

2.9.4.2 CDH Equations

It is desired to relate the velocity maneuvers at CDH to the observable, range rate. This will be measured at selected times before CDH.

From Equation 3.1 - 2 of Section 3.2

$$\dot{R}=[\dot{x}x + \dot{z}z]/R = \dot{x}\cos\eta + \dot{z}\sin\eta$$

Since at large ranges $\cos\eta \simeq 1$ and $\sin\eta \simeq 0$,

$$\dot{R} \simeq \dot{x} \qquad 4.2 - 1$$

Equations 4.1 - 8a, by defining

$$b = 2[2z_o + \dot{x}_o/\omega]$$

$$c = [2\dot{z}_o/\omega - x_o]$$

$$\rho = -[(\dot{z}_o/\omega)^2 - (2(\dot{x}_o/\omega)+3z_o)^2]$$

$$\gamma = \tan^{-1}[\dot{z}_o/(2\dot{x}_o + 3\omega z_o)]$$

can be written

$$x = c + \tfrac{3}{2}b\varphi - 2\rho\sin(\varphi+\gamma) \qquad \varphi = \omega t$$

$$z = b - \rho\cos(\varphi+\gamma)$$

Therefore,

$$\dot{x} = \tfrac{3}{2}b\dot{\varphi} - 2\rho\dot{\varphi}\cos(\varphi+\gamma)$$

$$\dot{z} = \rho\dot{\varphi}\sin(\varphi+\gamma) \qquad 4.2 - 2$$

For coellipticity, it is required

$$\dot{x} = \tfrac{3}{2}\omega z_o$$

$$\dot{z} = 0$$

So that the components of CDH at any point are

$$\Delta V_x = x_{REQ} - x_A = \tfrac{3}{2}\dot{\varphi}z - \tfrac{3}{2}\dot{\varphi}b + 2\rho\dot{\varphi}\cos(\varphi+\gamma)$$

$$= \tfrac{3}{2}\dot{\varphi}b - \tfrac{3}{2}\dot{\varphi}\rho\cos(\varphi+\gamma) - \tfrac{3}{2}b\dot{\varphi} + 2\rho\dot{\varphi}\cos(\varphi+\gamma) \qquad 4.2 - 3$$

$$= \tfrac{1}{2}\dot{\varphi}\rho\cos(\varphi+\gamma)$$

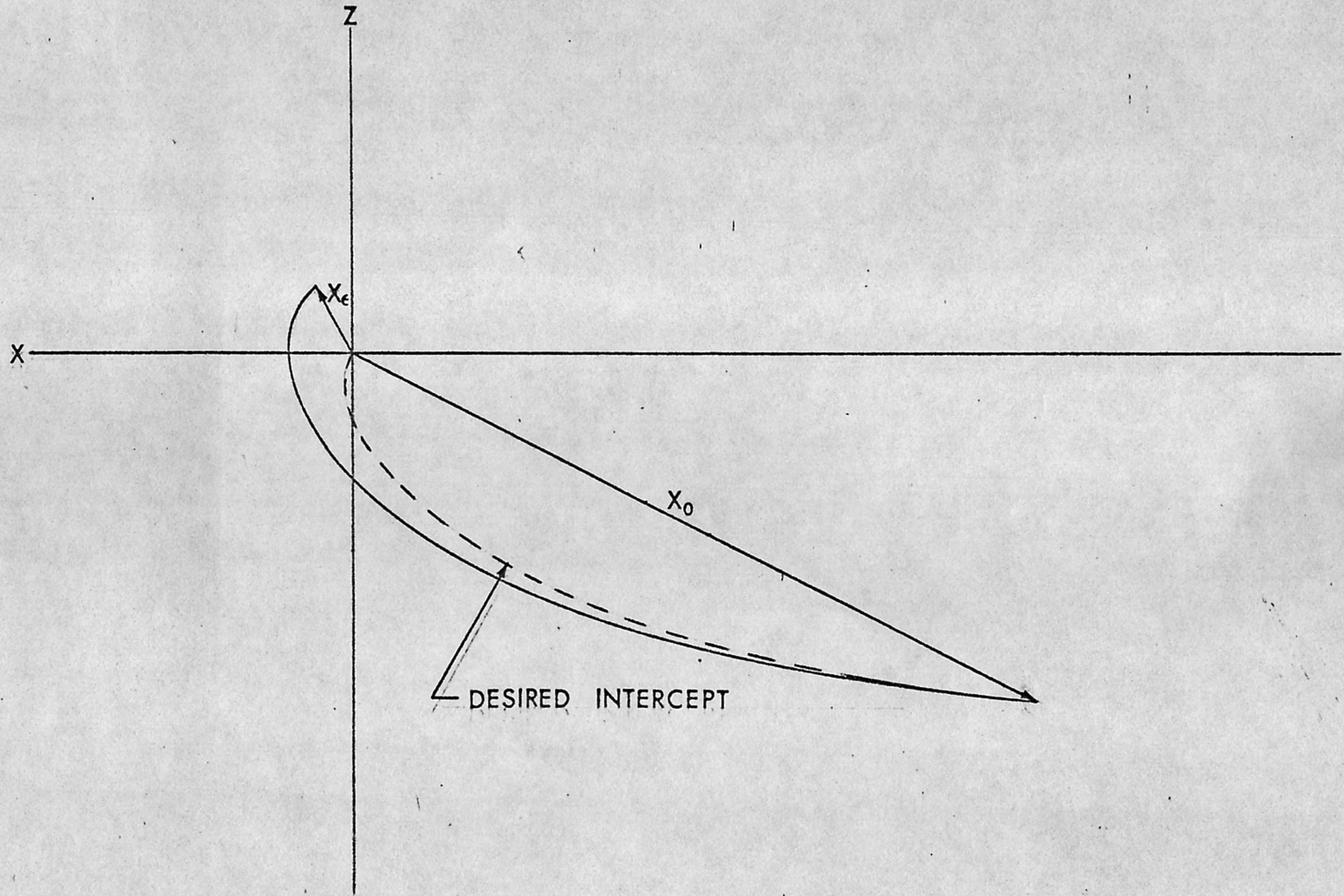


FIGURE VIII RENDEZVOUS PROBLEM

$$\Delta V_z = z_{REQ} - z_A = -\rho\dot{\varphi}\sin(\varphi+\gamma) \qquad 4.2. - 4$$

Taking the first equation of 4.2 - 2 and defining

$$\dot{R}_c = \tfrac{3}{2} b\dot{\varphi}$$

$$\dot{R}_m = -2\rho\dot{\varphi}$$

have

$$\dot{R} = \dot{R}_c + \dot{R}_m \sin(\varphi+\gamma)$$

as the equation to be solved. Taking t=0 at the first measurement noting that three will be required to determine $\dot{R}_c$, $\dot{R}_m$, γ :

$$\dot{R}_o = \dot{R}_c + \dot{R}_m \sin(\gamma)$$

$$\dot{R}_1 = \dot{R}_c + \dot{R}_m \sin(\varphi+\gamma)$$

$$\dot{R}_2 = \dot{R}_c + \dot{R}_m \sin(2\varphi+\gamma)$$

After considerable manipulation, the solution of this simultaneous set can be obtained as

$$\sin\gamma = (\dot{R}_o - \dot{R}_c)/\dot{R}_m$$

$$\dot{R}_c = \left[\dot{R}_o + \dot{R}_2 - 2\dot{R}_1 \cos\varphi\right]/2(1-\cos\varphi) \qquad \dot{R}_{E_o} = \dot{R}_o - \dot{R}_c$$

$$\dot{R}_m = \left[\dot{R}_{E_o}^2 + \dot{R}_{E_2}^2 - 2\dot{R}_{E_2}\dot{R}_{E_o}\cos 2\varphi\right]/\sin 2\varphi \qquad \dot{R}_{E_2} = \dot{R}_2 - \dot{R}_c$$

Therefore, if α is the elapsed central angle between the last measurement and the time of CDH, the maneuvers are given by

$$\Delta V_x = -\tfrac{1}{4}\dot{R}_m \cos(2\varphi+\gamma+\alpha)$$

$$\Delta V_z = \tfrac{1}{2}\dot{R}_m \sin(2\varphi+\gamma+\alpha)$$

2.9.4.3 Digital Computation of Transfer

In general, a transfer problem consists of finding the velocity maneuver required to go between given points subject to various constraints.

It is desired to solve the time of flight problem, in a manner appropriate for use with a digital computer.

As in figure VIII, let a Clohessy-Wiltshire (C-W) frame be attached to the passive particle, with ω, $\dot{X}_o$, X_o given. Then from the last section

$$X(t) = A(\omega t)X_o + B(\omega t)\dot{X}_o \qquad 4.3 - 1$$

Where $A(\omega t)$, $B(\omega t)$ are the C-W matrix functions of time. For intercept require $X(t) = 0$ after a time t:

$$A(\omega t)X_o + B(\omega t)\dot{X}_r = 0 \qquad 4.3 - 2$$

$$\dot{X}_r = -B^{-1}(\omega t)A(\omega t)X_o \qquad 4.3 - 3$$

Since maneuvers will be done in the active vehicle local vertical frame, compute

$$\begin{aligned}\dot{\vec{R}}_s &= \partial/\partial t(R_t + X_o) + \omega \times (R_t + X_o) \\ &= \dot{R}_t \hat{R}_t + \dot{X}_r + \omega \times (\vec{R}_t + X_r) \\ &= \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \dot{\vec{R}}_s \qquad \phi = \text{Central angle at intercept.}\end{aligned}$$

in the local vertical frame of the active vehicle.

If this $\dot{X}_r$ is applied, and the active vehicle precisely advanced along the resulting orbit, it will not, in general, intercept the origin. This is due to the approximate nature of the C-W equations, which results in an X_ϵ residual at the time for intercept. At this juncture, one may proceed in two similar but slightly different ways, both of which will be discussed.

1. Offset Targeting

Note that the general solution of the C-W equations for X_r is

$$\dot{X}_r^1 = B^{-1}(\omega t)X(t) - B^{-1}(\omega t)A(\omega t)X_o \qquad 4.3 - 4$$

In which $X(t)$ was set equal to zero for intercept. Since the solution obtained is known to miss by X_ϵ, retarget for $-X_\epsilon$ and reasonably expect to hit in between X_ϵ and $-X_\epsilon$, i.e., near the origin:

$$\dot{X}_r^2 = -B^{-1}(\omega t)X_\epsilon^1 - B^{-1}(\omega t)A(\omega t)X_o \qquad 4.3 - 5$$

This may again miss by X_ϵ^2 say, and a better solution may be obtained as before by aiming for $-X_\epsilon^2$:

$$\dot{X}_r^3 = -B^{-1}(\omega t)[X_\epsilon^1 + X_\epsilon^2] - B^{-1}(\omega t)A(\omega t)X_o \qquad 4.3 - 6$$

and so on

$$\dot{X}_r^n = -B^{-1}(\omega t) \sum_{i=0}^{n-1} X_\epsilon^i \quad -B^{-1}(\omega t)A(\omega t)X_o \qquad 4.3 - 7$$

where $X_\epsilon^0 = 0$

one may consider this an iteration on initial displacement by noting that the operation A(wt) transforms an initial displacement into a final displacement:

$$X_f = A(\omega t)X_i \qquad 4.3 - 8$$

thus targeting for $\sum X_\epsilon^i$ is the same as perturbing X_o by $A^{-1}(\omega t)\sum X_\epsilon^i$ and leads to the equation

$$\dot{X}_r^n = -B^{-1}(\omega t)A(\omega t)[X_o + A^{-1} \sum_{i=0}^{n-1} X_\epsilon^i] \qquad 4.3 - 9$$

where A^{-1} may be considered a matrix gain factor determining how an intercept error should perturb the initial displacement to effect convergence.

2. Successive C-W Frames

Attach a C-W frame to the active particle and imagine that at intercept, the passive vehicle has a position $-X_\epsilon$ in this frame. Then <u>in this frame</u> compute a velocity maneuver which would send a particle to this point from its origin in an $(\omega t)' = \Delta\nu_s$. Where $\Delta\nu_s$ is the change in the true anomaly of this new frame from transfer to intercept:

$$\dot{X}_\epsilon = -B^{-1}(\omega t)' X_\epsilon$$

Add this correction to the initial guess and proceed as before:

$$\dot{X}_r^n = -\sum_{i=0}^{n-1} B^{-1}(\omega t)_i' X_\epsilon^i \quad -B^{-1}(\omega t)A(\omega t)X_o$$

in effect constituting a new $\overline{\text{C-W}}$ frame at X_ϵ for each attempt, computing a solution to drive a particle to $-X_\epsilon$, and summing these.

Note that the propriety of these two methods lies in the tendency of the C-W equations to give exact results as the distance between the particles approaches zero.

2.9.4.5 CSI/CDH

Considering first the CDH maneuver, there are several ways to rigorously define coellipticity. The one in use for Apollo results in alignment of the semi-major axes and no variation in Δh to first order. In terms of the eccentric anomoly for each vehicle

$$R_1=a_1(1-e_1 \cos E_1)$$

$$R_2=a_2(1-e_2 \cos E_2)$$

therefore

$$\Delta h=R_2-R_1=a_2-a_1+a_1w_1 \cos E_1-a_2e_2 \cos E_2$$

$$=\Delta h_0+(a_1e_1 \cos E_1-a_2e_2 \cos E_2)$$

In order for there to be no variation in Δh, it must be true that $E_1=E_2-\phi$. (ϕ = phase angle) so that when the vehicle radius vectors are coincident $E_2=E_1$:

$$\Delta h_{CDH}=\Delta h_0+(a_1e_1-a_2e_2) \cos E_{CDH}$$

Then if $a_1e_1=a_2e_2$, h will be constant. It can be shown that for given true anomoly of both vehicles the same, the variation in Δh is the order of $e_1^2-e_2^2$.

For CSI, a straight iterative procedure is used wherein a trial velocity for a CSI is varied to compute a numerical partial derivative of change in η_{TPI} with respect to change in ΔV_{CSI}.

2.9.4.6 Multiple Linear Regression

Let an over-determined system of equations be given

$$RC = \Delta V + \epsilon$$

where R is an m x n m > n known matrix and $\vec{C}$ is an n-row by 1-column. Since the system is overdetermined, $\vec{\epsilon}$ will not in general by zero, hence let us seek to minimize its magnitude:

$$\vec{\epsilon} = R\vec{C} - \Delta\vec{V} \qquad 4.6 - 2$$

$$\epsilon^2 = \vec{\epsilon}^t\vec{\epsilon} = (\vec{C}^t R^t - \Delta\vec{V}^t)(R\vec{C} - \Delta\vec{V}) \qquad 4.6 - 3a$$

$$= \vec{C}^t R^t R\vec{C} - (\vec{C}^t R^t \Delta\vec{V} + \Delta\vec{V}^t R\vec{C}) + \Delta\vec{V}^t\Delta\vec{V} \qquad 4.6 - 3b$$

An extremum (hopefully minimum) of ϵ^2 will be found when the variation $\delta\epsilon^2$ consequent upon a variation δC is zero. i.e. require:

$$\delta\epsilon^2 = 2\left[\delta\vec{C}^t R^t R\vec{C} - \delta\vec{C}^t R^t \Delta\vec{V}\right] = 0 \qquad 4.6 - 4$$

since $\delta\Delta\vec{V} = \delta R = 0$. Now note that if α, a scalar is

$$\alpha = \vec{K}^t\vec{K} = (\vec{K}^t\vec{K})^t = \alpha^t$$

Since $\delta\epsilon^2$ is a scalar conclude that each term of 4.6 - 4 is scalar and substitute for $\vec{C}^t R^t R\delta\vec{C}$ and $\Delta\vec{V}^t R\delta\vec{C}$ their transposes.

Since the variations δC are arbitrary, we conclude that is minimum if

$$R^t R\vec{C} - R^t\Delta\vec{V} = 0$$

$$\vec{C} = (R^t R)^{-1} R^t \Delta\vec{V} \qquad 4.6 - 5$$

2.9.4.7 Digital Programs

Several digital programs have been developed for use in the generation and verification of backup charts and the study of the rendezvous problem. One of these, used for all of the above purposes, is a large multi-purpose Fortran program called Betelgeuse. It has the capability to integrate two vehicles through either Earth or Lunar orbit.

The program utilizes standardized input and output routines. As a normal course of events, approximately 40 different parameters are output at each integration step.

A set of subroutines are available which are designed to solve for each of the maneuvers required in the Concentric

Flight Plan. Included in these is the capability to execute externally supplied ΔV maneuvers. Taken together, these features provide a flexible tool for the study of rendezvous. Additionally, the program has the capability to generate up to 100 consecutive Monte Carlo runs, each with randomly dispersed initial conditions. This feature is used to make the runs which yield data for the construction of backup charts.

Another set of subroutines in the program represent a mechanization of the backup charts. These routines are capable of sampling data during a run, calculating, and applying the required maneuvers. In addition, there is a routine which executes braking and line-of-sight control during the final phase of the rendezvous. Random errors, biases, and drifts are included in each of these subroutines to make the simulation realistic. These routines are used in conjunction with the above mentioned Monte Carlo generator and a set of statistical analysis routines for performance analyses of backup charts.

A second Fortran program is used to generate the coefficients needed for CSI and CDH backup charts, which are based on Maclaurin expansions. The program takes data derived from Betelgeuse runs, and processes it using a multiple linear regression technique, yielding the required coefficients.

A third Fortran program is used to generate data for TPI and mid-course backup charts. It has all of the required equations mechanized, and outputs data which can be directly plotted to yield a backup chart.